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Cosmological bounds on light sterile neutrinos

(and ways around them)

Marco Cirelli (SPhT-CEA/Saclay & INFN)

with:

A.Strumia (Pisa)

Y.-Z. Chu (Yale)

G.Marandella (UC Davis)

F. Vissani (Gran Sasso)

based on:

NPB 708 (2005) 215, hep-ph/0403158 JCAP 12(2006)013, hep-ph/0607086 PRD 74 (2006) 085015, astro-ph/0608206

We want to study light sterile neutrinos $\nu_{\rm s}$.

 $\mathcal{O}(\mathrm{eV})$

- spin 1/2 fermions,
- neutral under SM forces,
- mix with active neutrinos.

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r-process nucleosynthesis pulsar kicks galactic ionization...

right-handed neutrino goldstino

- predicted in beyond SM models

axino majorino
dilatino
branino familino
modulino
mirror fermion...

The discovery of a new light particle would be fundamental.

Bounds on $\nu_{\rm s}$ from cosmology

Part 1: bounds from BBN

- T \sim MeV
- flavor is important
- matter effects in the plasma

Part 2: bounds from later cosmology

- $-\mathrm{\,T\,} \lesssim \mathrm{eV}$
- m_{ν} is important

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- $-\mathrm{T} \lesssim \mathrm{eV}$
- $m_
 u$ is important

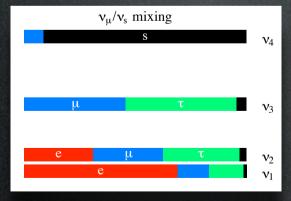
4 neutrino mixing

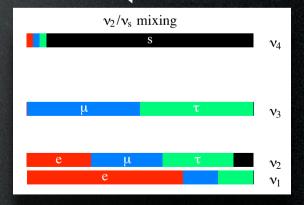
Instead of a limited 2ν formalism $u_l \to \cos\theta_{\rm s}
u_{l'} + \sin\theta_{\rm s}
u_{\rm s}$ we want a full 4ν formalism.

A simple parametrization: define a unit vector \vec{n} , which identifies a combination of active neutrinos

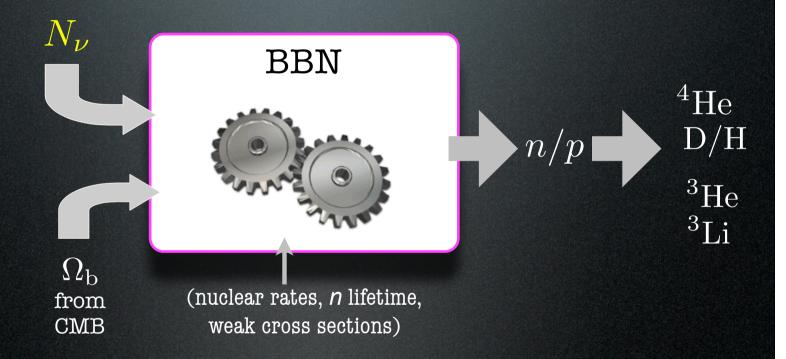
$$\begin{split} \vec{n}\cdot\vec{\nu} &= n_e\nu_e + n_\mu\nu_\mu + n_\tau\nu_\tau = n_1\nu_1 + n_2\nu_2 + n_3\nu_3 \\ \text{which mixes with } \nu_{\rm s} \text{ with an angle } \theta_{\rm S} \text{ ,} \\ \nu_{\rm s} \text{ has a mass } \mathbf{m}_4 \text{ .} \end{split}$$

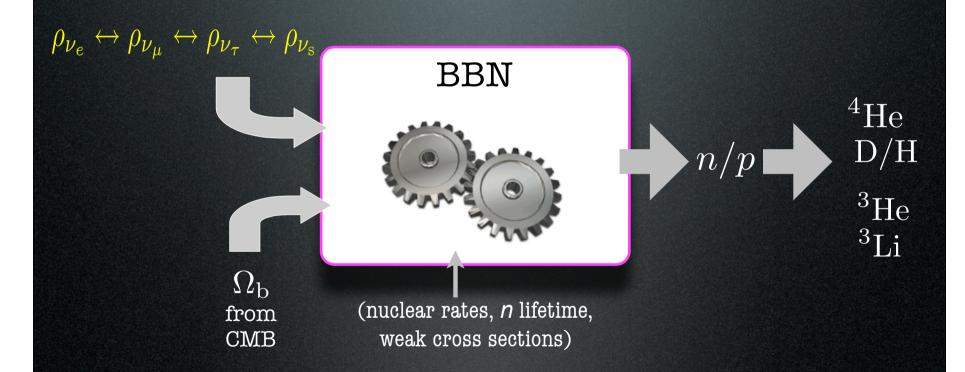
Basic cases: mixing with a flavor eigenstate, or a mass eigenstate

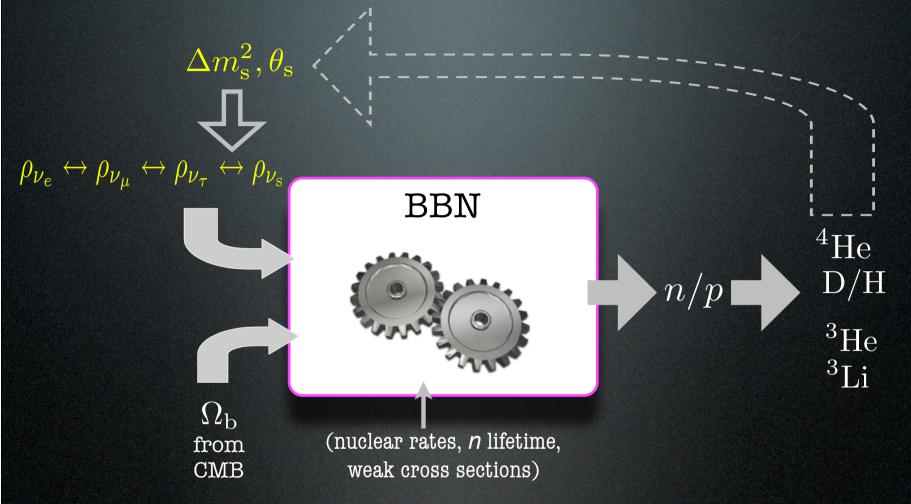




Free parameters: given a case, m_4 and $\theta_{
m S}$.







For any choice of $\Delta m_{\rm s}^2$, $\theta_{\rm s}$ a prediction from BBN.

For every choice of $\Delta m_{\rm s}^2$, $\theta_{\rm s}$, for T \gg MeV \longrightarrow 0.07 MeV follow:

1 kinetic equations for neutrino densities

$$ho_{
u_e},
ho_{
u_\mu},
ho_{
u_ au},
ho_{
u_{ extsf{s}}}$$

- 2 equation for n/p
- 3 equations of light nuclei (4He, D) production

Assumptions:

- no large lepton asymmetries
- neglect spectral distortions

1. Neutrino kinetic equations

4x4 neutrino density matrix ρ

3. scatterings and absorptions

$$\frac{d\rho}{dt} \equiv \frac{dT}{dt} \frac{d\rho}{dT} = -i \left[\mathcal{H}_m, \rho \right] - \left\{ \Gamma, (\rho - \rho^{\text{eq}}) \right\}$$

Dolgov, 1981 Barbieri, Dolgov 1990

2. oscillations

diag(1,1,1,0)

$$\mathcal{H}_{m} = \frac{1}{2E_{\nu}} \left[V \operatorname{diag}(m_{1}^{2}, m_{2}^{2}, m_{3}^{2}, m_{4}^{2}) V^{\dagger} + E_{\nu} \operatorname{diag}(V_{e}, V_{\mu}, V_{\tau}, 0) \right]$$

1.expansion

$$\left|\dot{T}\sim -H(T,
ho)T
ight|$$

Active/sterile mixing parameters

Hubble parameter depends on $ho_{
u_e} +
ho_{
u_\mu} +
ho_{
u_ au} +
ho_{
u_ au}$

$$H = (8\pi/3 \ G_N \ \rho_{\rm tot})^{1/2}$$

$$V_{e} = -\frac{199\sqrt{2}\pi^{2}}{180} \frac{\zeta(4)}{\zeta(3)} G_{F} \frac{T}{M_{W}^{2}} \left(T^{4} + \frac{1}{2} T_{\nu}^{4} \cos \theta_{W} \rho_{ee} \right)$$

$$V_{\mu} = -\frac{199\sqrt{2}\pi^{2}}{180} \frac{\zeta(4)}{\zeta(3)} G_{F} \frac{TT_{\nu}^{4}}{M_{W}^{2}} \left(\frac{1}{2} T_{\nu}^{4} \cos \theta_{W} \rho_{\mu\mu} \right)$$

$$V_{\tau} = -\frac{199\sqrt{2}\pi^{2}}{180} \frac{\zeta(4)}{\zeta(3)} G_{F} \frac{TT_{\nu}^{4}}{M_{W}^{2}} \left(\frac{1}{2} T_{\nu}^{4} \cos \theta_{W} \rho_{\tau\tau} \right)$$

$$V_{s} = 0$$

v thermal masses

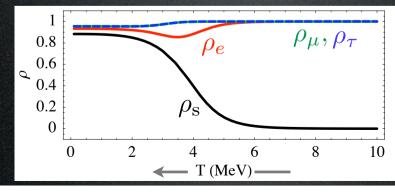
1. Neutrino kinetic equations

What happens qualitatively:

- for $T\gg {
 m MeV}$, matter effects suppress mixing
- $(\rho_{\nu_{\rm s}} \simeq 0)$
- as T decreases, at a certain point oscillations $\nu_{
 m active} \leftrightarrow \nu_{
 m s}$ can begin $(\Delta m_{
 m s}^2, \theta_{
 m s})$

 $(\rho_{\nu_{\rm s}} \nearrow)$

- + redistribution $\nu_{\mathrm{active}} \leftrightarrow \nu_{\mathrm{active}}$
- meanwhile: ν decouple at $T \sim {\rm MeV}, e^+e^-$ annihilate...
- Output: $\rho_{\nu_e}(T), \rho_{\nu_{\mu}}(T), \rho_{\nu_{\tau}}(T), \rho_{\nu_{s}}(T)$

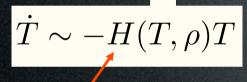


$$\nu_e/\nu_s$$
 mixing
$$\Delta m_s^2 = 6 \, 10^{-4} \, \text{eV}^2$$

$$\tan^2 2\theta_s = 2 \, 10^{-1}$$

2. n/p ratio

$$\dot{r} \equiv \frac{dT}{dt}\frac{dr}{dT} = \Gamma_{p\to n}(1-r) - r\Gamma_{n\to p} \qquad r = \frac{n_n}{n_n + n_p}$$



Hubble parameter depends on $ho_{
u_e} +
ho_{
u_\mu} +
ho_{
u_ au} +
ho_{
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weak interactions

$$n \longleftrightarrow p + e^{-} + \bar{\nu}_{e}$$

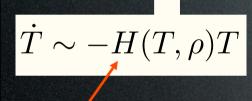
$$n + \nu_{e} \longleftrightarrow p + e^{-}$$

$$n + e^{+} \longleftrightarrow p + \bar{\nu}_{e}.$$

depend on $\,
ho_{
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depend on $\,
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So, where does a ν_s enter the game?

- (A) total energy density \Rightarrow expansion parameter
- (B) depletion of ν_e density \Rightarrow weak rates

3. Light elements production

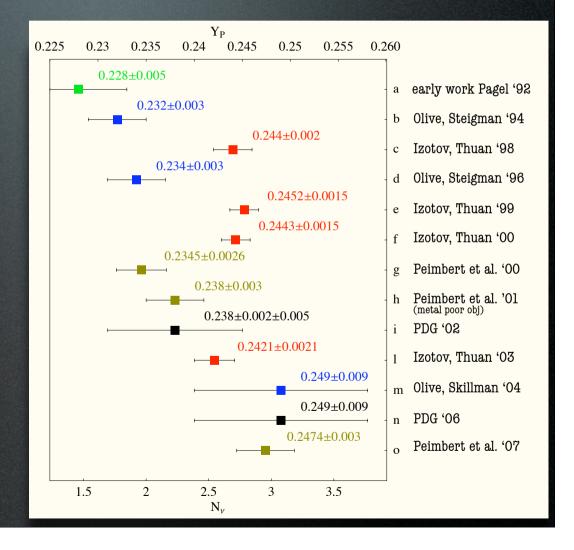
A network of Boltzmann equations with up-to-date nuclear rates...

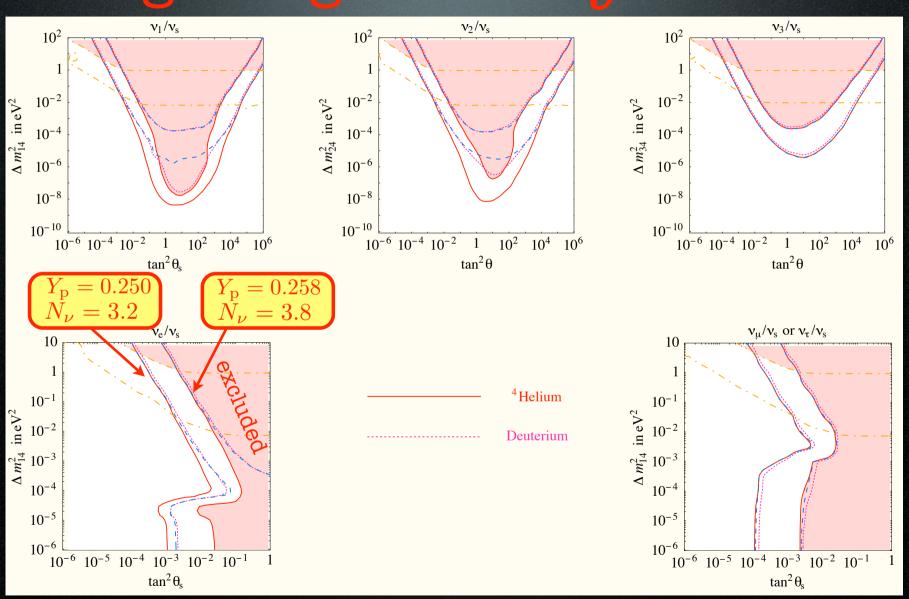
4. Observations

Determinations of primordial ⁴He are somehow controversial.

Conservatively, take $Y_{
m p}=0.249\pm0.009$

(Determinations of D/H are currently less useful.)



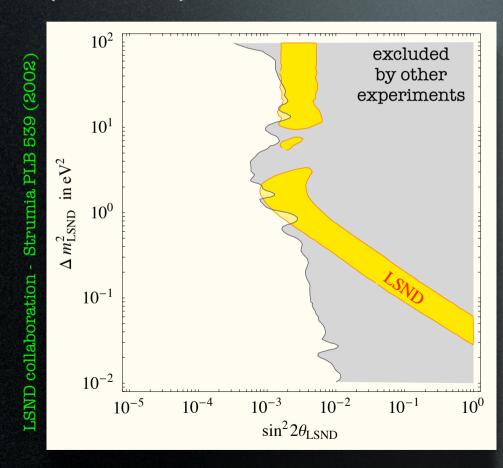


LSND

LSND claims evidence for $ar{
u}_{\mu}
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u}_{e}$ with $\Delta m^{2}
eq \Delta m_{ ext{sun,}}^{2}$

Requires a new (sterile) neutrino: $ar
u_{\mu}
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u_{
m s}
ightarrow ar
u_{
m e}$

(if oscillations)



with mixing $\vec{n} \simeq (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0)$ i.e. $\theta_{es}\theta_{\mu s} \simeq \theta_{LSND}$

$$\Delta m_{\rm LSND}^2 \simeq 1 \text{ eV}^2$$

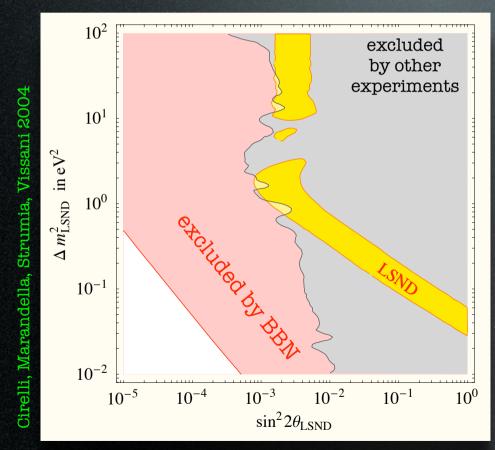
 $\sin^2 2\theta_{\rm LSND} \simeq 10^{-3}$

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Bottom Line:

BBN excludes the LSND $u_{
m S}$ (too much cosmo expansion)

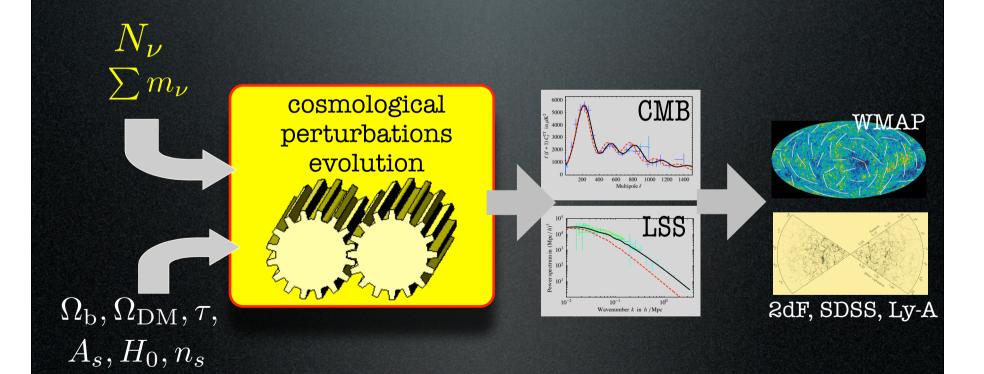
Bounds on $\nu_{\rm s}$ from cosmology

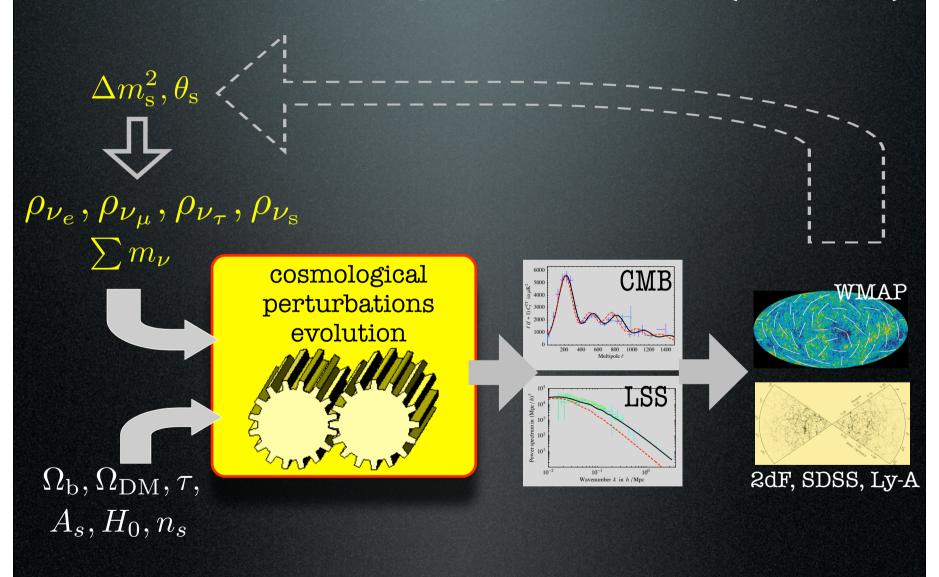
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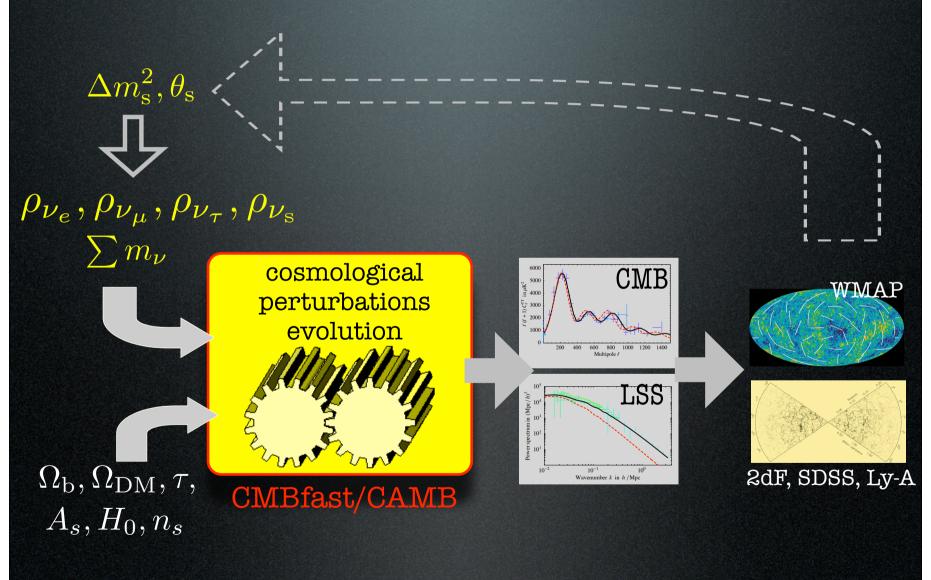
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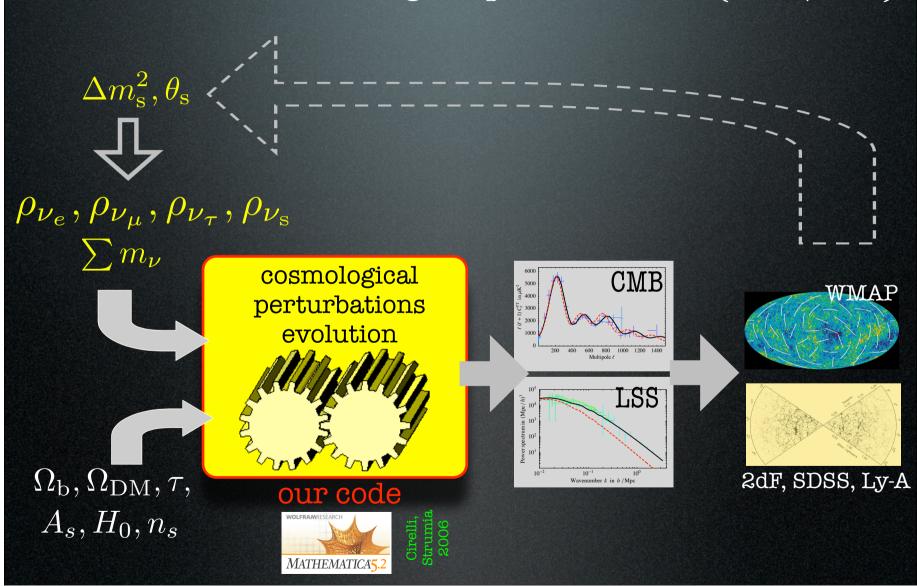
Part 2: bounds from later cosmology

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- $m_
 u$ is important





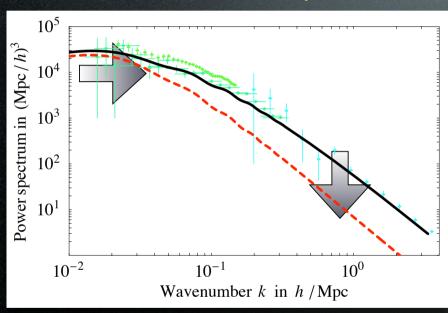




Neutrinos affect cosmological perturbations (CMB, LSS).

Neutrino free-streaming suppresses the growth of LSS on small scales:

(more precisely: massive neutrinos contribute to the energy density of the Universe during MD but they don't source in the Newton equation for $\delta_{\rm dm}$)



$$k_{\rm NR} = 0.018 \ \Omega_{\rm m}^{-1/2} \left(\frac{\sum m_{\nu}}{\rm eV}\right)^{1/2} h_0 \ {\rm Mpc}^{-1}$$

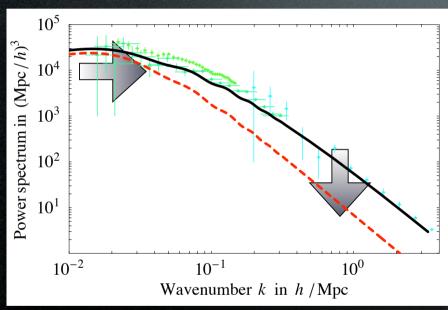
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a bound on $\sum m_{
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$$\sum m_{\nu_i} < 0.40 \text{ eV}$$

(@ 99.9% C.L., global fit)

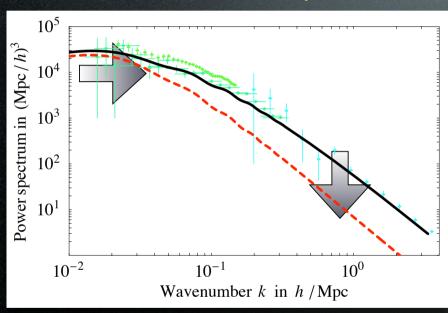
Cirelli, Strumia 2006

in presence of $\rho_{\nu_e}, \rho_{\nu_\mu}, \rho_{\nu_\tau}, \rho_{\nu_s}$: $\sum m_i \rho_i < 0.40 \text{ eV}$

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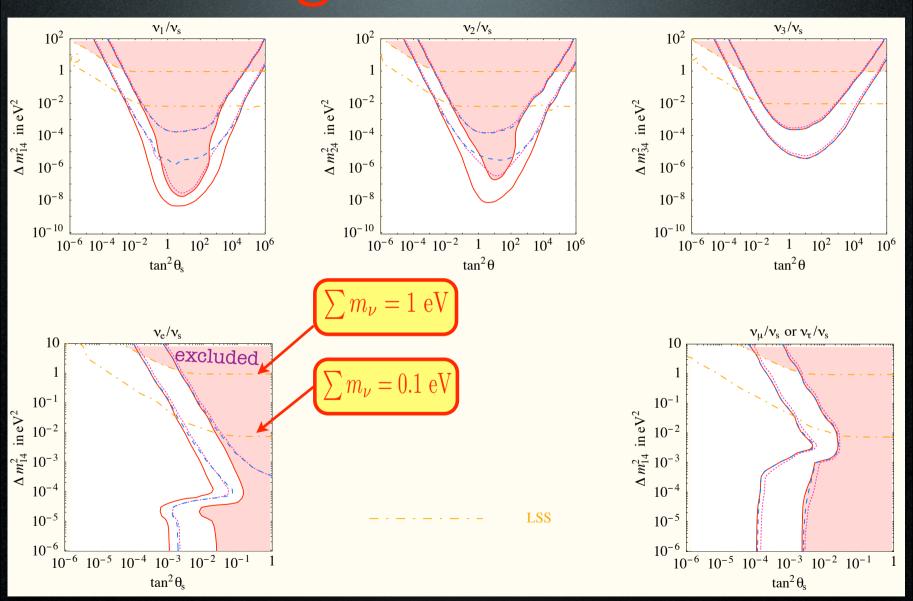
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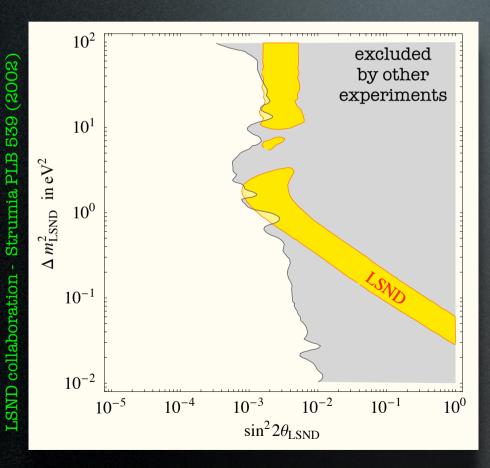
in presence of $\rho_{\nu_e}, \rho_{\nu_\mu}, \rho_{\nu_\tau}, \rho_{\nu_s}$: $\sum m_i \rho_i < 0.40 \text{ eV}$

 $u_{\rm s}$ contribute to $\sum m_{
u} \implies {\rm a~bound~on}~m_4$ i.e. $\Delta m_{\rm s}^2$



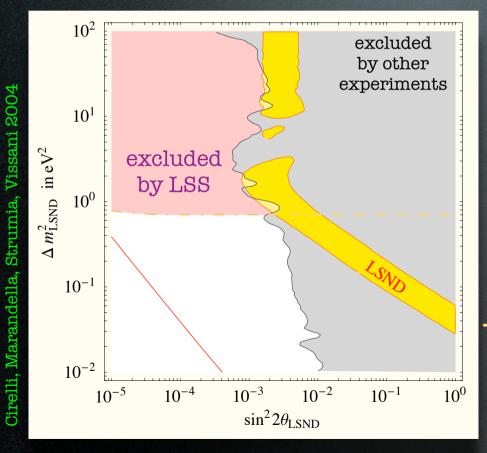
Cirelli, Marandella, Strumia, Vissani 2004

LSND



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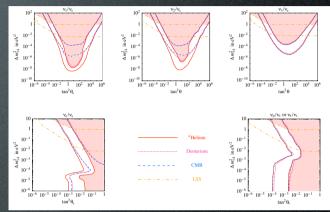
Bottom Line:

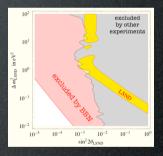
LSS excludes the LSND $u_{
m S}$ (too much $\sum m_{
u}$)

Recap

- ullet Cosmology gives some of the most stringent bounds on $u_{
 m s}$:
 - BBN constraints the total $N_{
 u}$ and the depletion of $u_e, \bar{
 u}_e,$
 - LSS constraints $\sum m_{
 u}$.

ullet BBN and LSS reject the LSND $u_{
m S}$:





Non-standard modifications

A. a large primordial lepton asymmetry

B. neutrino interactions with new light particles

C. low reheating temperature

D. ...

[skip to conclusions]

Non-standard modifications

A. a large primordial lepton asymmetry

$$L_{\nu} = \frac{n_{\nu} - n_{\bar{\nu}}}{n_{\gamma}}$$

Foot, Volkas PRL 75 (1995) P.Di Bari (2002, 2003) V.Barger et al., PLB 569 (2003)

An asymmetry $L_{\nu} \approx \eta = 6 \ 10^{-10}$ (baryon asym.) would be natural, but a priori $L_{\nu} \sim \mathcal{O}(10^{-2})$ is possible.

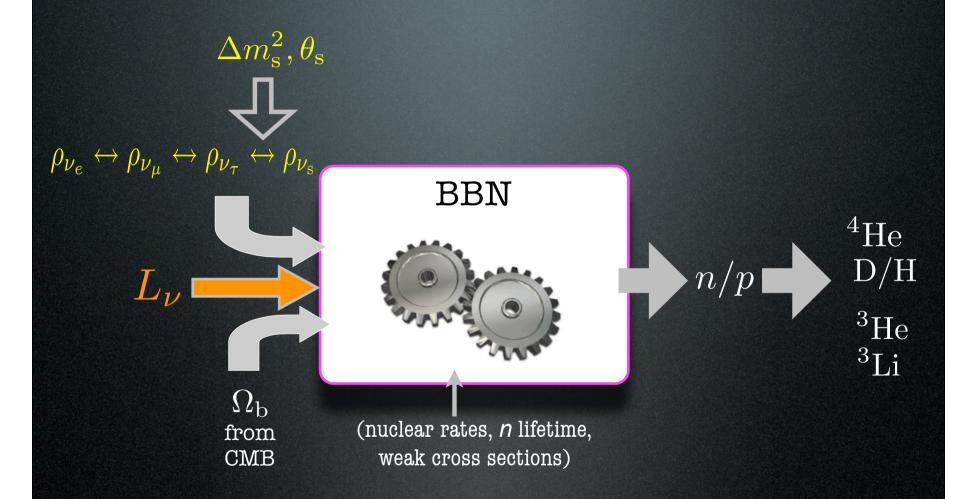
Dolgov,..., Semikoz (2002)
Abazajian, Beacom, Bell (2002)

Abazajian, Beacom, Bell (2002) Cuoco,..., Serpico (2004) Serpico, Raffelt (2005)

B. neutrino interactions with new light particles

C. low reheating temperature

BBN with lepton asymmetry



For any choice of $\Delta m_{\rm s}^2$, $\theta_{\rm s}$, L_{ν} a prediction from BBN.

BBN with lepton asymmetry

- follow separately / and /
- an extra term in the neutrino matter potentials

$$\frac{d\rho}{dt} \equiv \frac{dT}{dt} \frac{d\rho}{dT} = -i \left[\mathcal{H}_m, \rho \right] - \left\{ \Gamma, (\rho - \rho^{\text{eq}}) \right\}$$
3. scatterings and absorptions



2. oscillations

$$\mathcal{H}_{m} = \frac{1}{2E_{\nu}} \left[V \operatorname{diag}(m_{1}^{2}, m_{2}^{2}, m_{3}^{2}, m_{4}^{2}) V^{\dagger} + E_{\nu} \operatorname{diag}(V_{e}, V_{\mu}, V_{\tau}, 0) \right]$$

1.expansion

$$\dot{T} \sim -H(T, \rho)T$$

$$V_{e} \simeq \left[\pm \sqrt{2}G_{F}n_{\gamma} \left[\frac{1}{2}\eta + 2L_{\nu_{e}} + L_{\nu_{\mu}} + L_{\nu_{\tau}} \right] - \frac{199\sqrt{2}\pi^{2}}{180} \frac{\zeta(4)}{\zeta(3)} G_{F} \frac{T_{\nu}}{M_{W}^{2}} \left[T^{4} + \frac{1}{4}T_{\nu}^{4} \cos^{2}\theta_{w}(\rho_{ee} + \bar{\rho}_{ee}) \right] \right]$$

$$V_{\mu} \simeq \left[\pm \sqrt{2}G_{F}n_{\gamma} \left[\frac{1}{2}\eta + L_{\nu_{e}} + 2L_{\nu_{\mu}} + L_{\nu_{\tau}} \right] - \frac{199\sqrt{2}\pi^{2}}{180} \frac{\zeta(4)}{\zeta(3)} G_{F} \frac{T_{\nu}T^{4}}{M_{W}^{2}} \left[\frac{1}{4}T_{\nu}^{4} \cos^{2}\theta_{w}(\rho_{\mu\mu} + \bar{\rho}_{\mu\mu}) \right] \right]$$

$$V_{\tau} \simeq \left[\pm \sqrt{2}G_{F}n_{\gamma} \left[\frac{1}{2}\eta + L_{\nu_{e}} + L_{\nu_{\mu}} + 2L_{\nu_{\tau}} \right] - \frac{199\sqrt{2}\pi^{2}}{180} \frac{\zeta(4)}{\zeta(3)} G_{F} \frac{T_{\nu}T^{4}}{M_{W}^{2}} \left[\frac{1}{4}T_{\nu}^{4} \cos^{2}\theta_{w}(\rho_{\tau\tau} + \bar{\rho}_{\tau\tau}) \right] \right]$$

$$V_{s} = 0$$

v thermal masses

BBN with lepton asymmetry

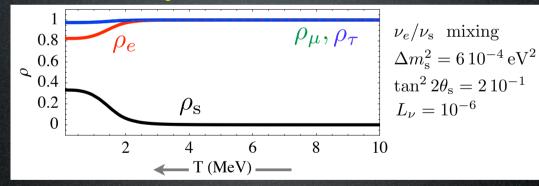
What happens qualitatively:

- for $T\gg {
 m MeV}$, matter effects suppress mixing
- $(\rho_{\nu_{\rm s}} \simeq 0)$
- despite T decreasing, the asymmetry term inhibits $\nu_{
 m active} \leftrightarrow \nu_{
 m s}$ oscillations



- $\nu_{\rm s}$ are less efficiently produced (or not at all)

 $(\rho_{\nu_{\rm s}}\ll 1)$



comparison with standard casel

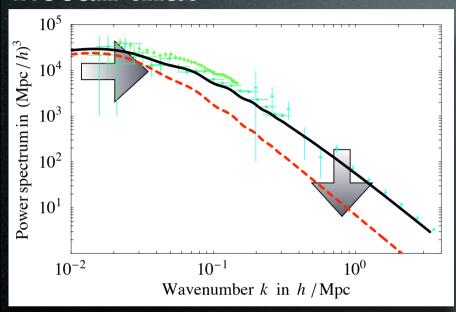
- (also: n/p weak rates affected by $ho_{
u_e}
eq ar{
ho}_{
u_e}$)

Assumptions:

- $L_{
 u_e} = L_{
 u_\mu} = L_{
 u_ au}$ for simplicity
- ullet non-dynamical $L_
 u$
- neglect spectral distortions

LSS with lepton asymmetry

Recall that



$$\Rightarrow \sum m_{\nu_i} < 0.40 \text{ eV}$$

(@ 99.9% C.L., global fit)

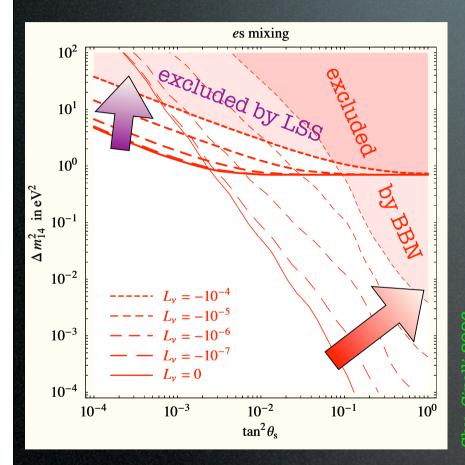
or better

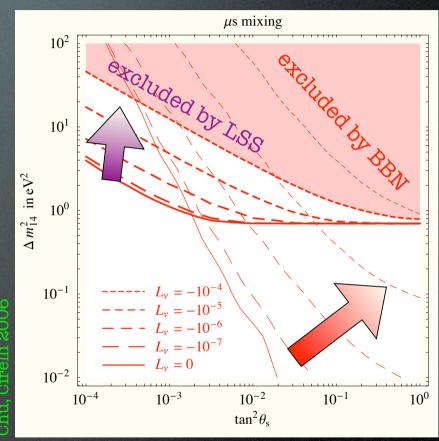
 $\sum m_i \rho_i < 0.40 \text{ eV}$

 $L_{
u}$ suppresses $u_{
m s}$ production $(
ho_{
u_{
m s}} \ll 1)$:
the bound on m_4 i.e. $\Delta m_{
m s}^2$ is relaxed.

with lepton asymmetry

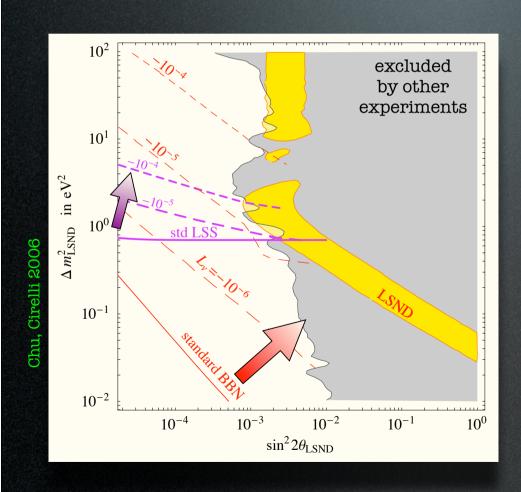
Portions of the parameter space are reopened:





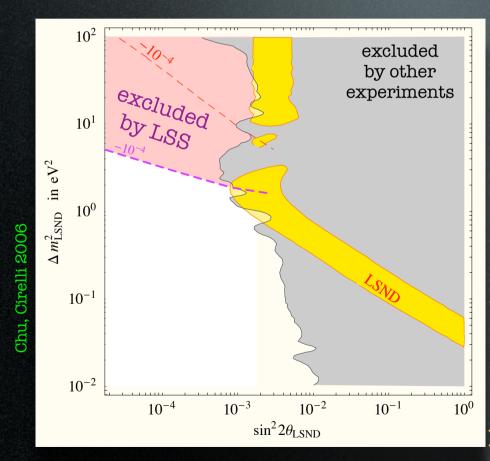
LSND with lepton asymmetry

Portions of the parameter space are reopened:



LSND with lepton asymmetry

Portions of the parameter space are reopened:



Bottom Line:

postulating a primordial asymmetry $L_{
u} \simeq -10^{-4}$ reconciles LSND and cosmology

Non-standard modifications

A. a large primordial lepton asymmetry

B. neutrino interactions with new light particles

C. low reheating temperature

D. ...

[skip to conclusions]

Non-standard modifications

A. a large primordial lepton asymmetry

B. neutrino interactions with new light particles couplings $g \nu \bar{\nu} \phi$ mediate neutrino decay at late times: neutrinos disappear \Rightarrow not subject to cosmo bounds

"Neutrinoless Universe", Beacom, Bell, Dodelson (2004)

also for sterile neutrinos $g \nu_s \bar{\nu} \phi$ "LSND", Palomares-Ruiz, Pascoli, Schwetz (2005) in general, interacting neutrinos pop up often

"MaVaNs", Fardon, Nelson, Weiner (2004)
"Late-time masses", Chacko, Hall et al., (2004)

C. low reheating temperature D. ...

 $u \leftrightarrow \phi$ couplings imply a tightly coupled fluid at recombination for $g > 10^{-8}, 10^{-14}$ (decay, scattering) Hannestad, Raffelt (2005) \Rightarrow neutrino free streaming is obstructed

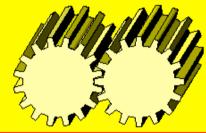
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 $N_
u^{
m norm}, N_
u^{
m int}, N_\phi, m_
u, m_\phi$

Boltzmann eqs for tightly coupled fluid

standard ν eqs

cosmological perturbations evolution

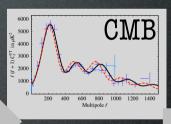


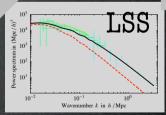
 $\Omega_{\mathrm{b}}, \Omega_{\mathrm{DM}}, au, A_{s}, H_{0}, n_{s}$

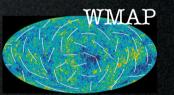








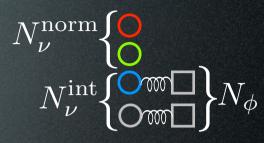


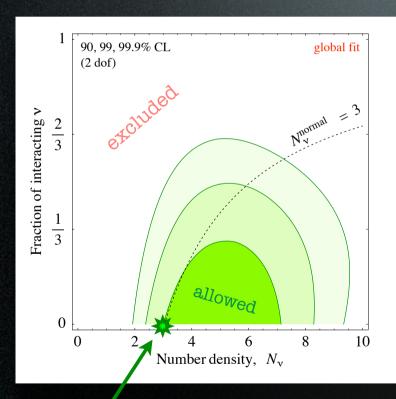


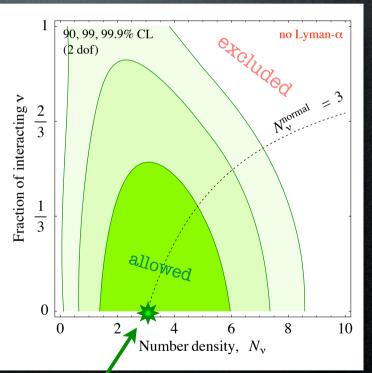


2dF, SDSS, Ly-A

Case: $N_{
u}^{
m norm}$ standard neutrinos, $N_{
u}^{
m int}$ interacting with N_{ϕ} scalars, everything massless.



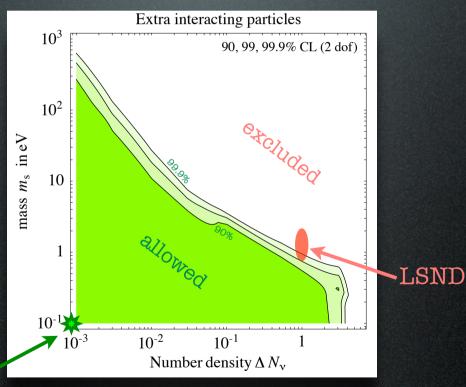




Cirelli, Strumia (2006)

Case: three standard neutrinos (massless), $\Delta N_{
u}$ interacting sterile neutrinos, with mass $m_{\rm s}$.

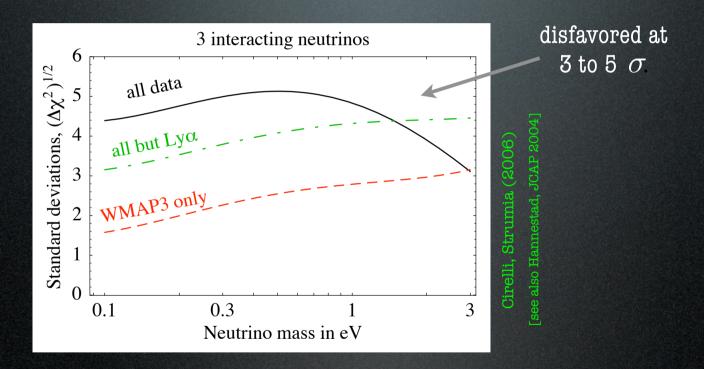




Standard cosmology

Case: three massive neutrinos, interacting with a massless scalar.



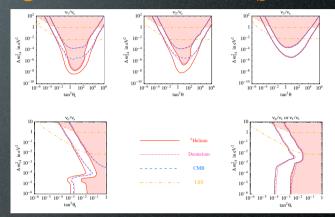


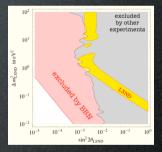
Bottom Line: Cosmology disfavors, at various degrees, interacting (non-freely streaming) neutrinos.

Conclusions

- Cosmology gives some of the most stringent bounds on ν_s :
 - BBN constraints the total $N_{
 u}$ and the depletion of $u_e, \bar{
 u}_e$
 - LSS constraints $\sum m_{
 u}$

ullet BBN and LSS reject the LSND $u_{\rm S}$:





- ullet a large lepton asymmetry relaxes BBN and LSS bounds $(L_{
 u} \simeq -10^{-4} \; ext{to reconcile LSND})$
- interacting neutrinos may avoid some bounds, but look problematic (lacking free streaming)

Extra Slides

Dodelson's (Chicago, 2003) notations

$$\dot{\Theta}+ik\mu\Theta=-\dot{\Phi}-ik\mu\Psi-\dot{ au}igl[\Theta_{0}-\Theta+\mu v_{\mathrm{b}}-1/2\,\mathcal{P}_{2}(\mu)\Piigr]\ \dot{ au}=d au/d\eta=-n_{e}\sigma_{T}a$$
 $\Pi=\Theta_{2}+\Theta_{P2}+\Theta_{P0}$ $\dot{\Theta}_{P}+ik\mu\Theta_{P}=-\dot{ au}igl[\Theta_{P}+1/2igl(1-\mathcal{P}_{2}(\mu)igr)\Piigr]$

$$\left\{ egin{aligned} \dot{\delta}_{
m dm} + ikv_{
m dm} &= -3\dot{\Phi} \ \dot{v}_{
m dm} + rac{\dot{a}}{a}v_{
m dm} &= -ik\Psi \end{aligned}
ight\} {
m dark\ matter}$$

$$egin{aligned} \dot{\delta}_{
m b}+ikv_{
m b}&=-3\dot{\Phi} & _{R=3
ho_{
m b}^0/4
ho_{\gamma}^0}\ \dot{v}_{
m b}+rac{\dot{a}}{a}v_{
m b}&=-ik\Psi+rac{\dot{ au}}{R}ig[v_{
m b}+3i\Theta_1ig] \end{aligned} egin{aligned} ext{baryons} \end{aligned}$$

$$\dot{\mathcal{N}}+irac{q_{
u}}{E_{
u}}k\mu\mathcal{N}=-\dot{\Phi}-irac{E_{
u}}{q_{
u}}k\mu\Psi\left\} \ {
m neutrinos} \ {$$

CMB Power spectrum

$$C_{\ell} \propto \int dk [\ldots] \Theta_{\ell}(k)$$

$$P(k) \propto \langle \delta_{
m m}(k)^2
angle$$

$$k^{2}\Phi + 3\frac{\dot{a}}{a}\left(\dot{\Phi} - \Psi\frac{\dot{a}}{a}\right) = 4\pi G_{N}a^{2}\left[\rho_{\mathrm{m}}\delta_{\mathrm{m}} + 4\rho_{\mathrm{r}}\delta_{\mathrm{r}}\right]$$

$$k^{2}\left(\Phi + \Psi\right) = -32\pi G_{N}a^{2}\rho_{\mathrm{r}}\Theta_{\mathrm{r},2}$$
metric

$$\dot{\Theta} + ik\mu\Theta = -\dot{\Phi} - ik\mu\Psi - \dot{\tau}\left[\Theta_0 - \Theta + \mu v_{\mathrm{b}} - 1/2\,\mathcal{P}_2(\mu)\Pi\right] \ \dot{\tau} = d\tau/d\eta = -n_e\sigma_T a \quad \Pi = \Theta_2 + \Theta_{P2} + \Theta_{P0}$$

$$\Theta = \frac{\delta T}{T} \qquad f(\vec{x}, \vec{p}) = \frac{1}{e^{\frac{p}{T + \delta T}} - 1}$$

Fourier:
$$\Theta(\vec{x}, \vec{p}, t) \longrightarrow \Theta(k, \mu, \eta)$$
 $\mu = \hat{k} \cdot \hat{p}$

Expand in multipoles:

$$\Theta_{\ell}(k,\eta) = \frac{1}{(-1)^{\ell}} \int_{-1}^{1} d\mu \frac{1}{2} \mathcal{P}(\mu) \Theta(k,\mu,\eta)$$

$$\dot{v}_{
m b}+rac{\dot{a}}{a}v_{
m b}=-ik\Psi+rac{\dot{ au}}{R}ig[v_{
m b}+3i\Theta_1ig]$$
 baryons $\dot{\mathcal{N}}+irac{q_
u}{E_
u}k\mu\mathcal{N}=-\dot{\Phi}-irac{E_
u}{q_
u}k\mu\Psiig\}$ neutrinos

CMB Power spectrum

$$C_{\ell} \propto \int dk [\ldots] \Theta_{\ell}(k)$$

$$P(k) \propto \langle \delta_{
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$$k^2\Phi + 3\frac{\dot{a}}{a}\left(\dot{\Phi} - \Psi\frac{\dot{a}}{a}\right) = 4\pi G_N a^2 \left[\rho_{\rm m}\delta_{\rm m} + 4\rho_{\rm r}\delta_{\rm r}\right]$$

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$$\dot{\Theta} + ik\mu\Theta = -\dot{\Phi} - ik\mu\Psi - \dot{\tau} \left[\Theta_0 - \Theta + \mu v_{\rm b} - 1/2 \,\mathcal{P}_2(\mu)\Pi\right] \\ \dot{\tau} = d\tau/d\eta = -n_e\sigma_T a \qquad \Pi = \Theta_2 + \Theta_{P2} + \Theta_{P0} \\ \dot{\Theta}_P + ik\mu\Theta_P = -\dot{\tau} \left[\Theta_P + 1/2 \left(1 - \mathcal{P}_2(\mu)\right)\Pi\right]$$
 photons

$$\left\{ \dot{\delta}_{
m dm} + ikv_{
m dm} = -3\dot{\Phi} \ \dot{v}_{
m dm} + \frac{\dot{a}}{a}v_{
m dm} = -ik\Psi
ight\} {
m dark\ matter}$$

$$\delta_{\mathrm{dm}} = \frac{\delta \rho_{\mathrm{dm}}}{\rho_{\mathrm{dm}}} \qquad \rho_{\mathrm{dm}}(\vec{x}, t) = \rho_{\mathrm{dm}}^{0} \left(1 + \delta_{\mathrm{dm}}(\vec{x}, t) \right)$$

and velocity $v_{
m dm}$

Fourier: $\delta_{\mathrm{dm}}(\vec{x},t) \longrightarrow \delta_{\mathrm{dm}}(k,\eta)$ $v_{\mathrm{dm}}(\vec{x},t) \longrightarrow v_{\mathrm{dm}}(k,\eta)$

CMB Power spectrum

$$C_{\ell} \propto \int dk [\ldots] \Theta_{\ell}(k)$$

$$P(k) \propto \langle \delta_{
m m}(k)^2
angle$$

$$k^{2}\Phi + 3\frac{\dot{a}}{a}\left(\dot{\Phi} - \Psi\frac{\dot{a}}{a}\right) = 4\pi G_{N}a^{2}\left[\rho_{\mathrm{m}}\delta_{\mathrm{m}} + 4\rho_{\mathrm{r}}\delta_{\mathrm{r}}\right]$$

$$k^{2}\left(\Phi + \Psi\right) = -32\pi G_{N}a^{2}\rho_{\mathrm{r}}\Theta_{\mathrm{r},2}$$
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$$\dot{\Theta} + ik\mu\Theta = -\dot{\Phi} - ik\mu\Psi - \dot{\tau} \left[\Theta_{0} - \Theta + \mu v_{b} - 1/2 \,\mathcal{P}_{2}(\mu)\Pi\right]
\dot{\tau} = d\tau/d\eta = -n_{e}\sigma_{T}a \qquad \Pi = \Theta_{2} + \Theta_{P2} + \Theta_{P0}
\dot{\Theta}_{P} + ik\mu\Theta_{P} = -\dot{\tau} \left[\Theta_{P} + 1/2\left(1 - \mathcal{P}_{2}(\mu)\right)\Pi\right]$$
photons

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m b}^0/4
ho_{\gamma}^0}\ \dot{v}_{
m b}+rac{\dot{a}}{a}v_{
m b}&=-ik\Psi+rac{\dot{ au}}{R}ig[v_{
m b}+3i\Theta_1ig] \end{aligned} egin{aligned} ext{baryons} \end{aligned}$$

$$egin{array}{lll} \delta_{
m b}(k,\eta) & \qquad & ext{Thomson scattering} \ v_{
m b}(k,\eta) & & e^-\gamma & & e^-\gamma \end{array}$$

CMB Power spectrum

$$C_{\ell} \propto \int dk [\ldots] \Theta_{\ell}(k)$$

$$P(k) \propto \langle \delta_{
m m}(k)^2 \rangle$$

$$k^{2}\Phi + 3\frac{\dot{a}}{a}\left(\dot{\Phi} - \Psi\frac{\dot{a}}{a}\right) = 4\pi G_{N}a^{2}\left[\rho_{\mathrm{m}}\delta_{\mathrm{m}} + 4\rho_{\mathrm{r}}\delta_{\mathrm{r}}\right]$$

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$$\dot{\Theta} + ik\mu\Theta = -\dot{\Phi} - ik\mu\Psi - \dot{\tau} \left[\Theta_0 - \Theta + \mu v_{\rm b} - 1/2 \,\mathcal{P}_2(\mu)\Pi\right]
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photons

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m dm} + ikv_{
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m dm} + rac{\dot{a}}{a}v_{
m dm} &= -ik\Psi \end{aligned}
ight\} {
m dark\ matter}$$

$$\delta_{
m h}$$
 scalar metric perturbations: $g_{\mu
u} = \eta_{\mu
u} + \delta \eta_{\mu
u} (\Psi, \Phi)$ $\dot{v}_{
m h}$ $g_{\mu
u} = \left(egin{array}{cccc} -1 - 2\Psi & 0 & 0 & 0 & 0 \ 0 & a^2(1+2\Phi) & 0 & 0 & 0 \ 0 & 0 & a^2(1+2\Phi) & 0 & 0 \ 0 & 0 & a^2(1+2\Phi) & 0 \end{array}
ight)$

Fourier: $\Psi(\vec{x},t) \longrightarrow \Psi(k,\eta)$ $\Phi(\vec{x},t) \longrightarrow \Phi(k,\eta)$

CMB Power spectrum

$$C_{\ell} \propto \int dk [\ldots] \Theta_{\ell}(k)$$

$$P(k) \propto \langle \delta_{
m m}(k)^2
angle$$

$$k^2\Phi + 3\frac{\dot{a}}{a}\left(\dot{\Phi} - \Psi\frac{\dot{a}}{a}\right) = 4\pi G_N a^2 \left[\rho_{\rm m}\delta_{\rm m} + 4\rho_{\rm r}\delta_{\rm r}\right]$$

$$k^2\left(\Phi + \Psi\right) = -32\pi G_N a^2 \rho_{\rm r}\Theta_{\rm r,2}$$
metric

$$\dot{\Theta} + ik\mu\Theta = -\dot{\Phi} - ik\mu\Psi - \dot{\tau} \left[\Theta_0 - \Theta + \mu v_{\rm b} - 1/2 \,\mathcal{P}_2(\mu)\Pi\right] \\ \dot{\tau} = d\tau/d\eta = -n_e\sigma_T a \qquad \Pi = \Theta_2 + \Theta_{P2} + \Theta_{P0} \\ \dot{\Theta}_P + ik\mu\Theta_P = -\dot{\tau} \left[\Theta_P + 1/2 \left(1 - \mathcal{P}_2(\mu)\right)\Pi\right]$$
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m b}+rac{\dot{a}}{a}v_{
m b}&=-ik\Psi+rac{\dot{ au}}{R}ig[v_{
m b}+3i\Theta_1ig] \end{aligned} } ext{ baryons}$$

$$\dot{\mathcal{N}} + i \frac{q_{\nu}}{E_{\nu}} k \mu \mathcal{N} = -\dot{\Phi} - i \frac{E_{\nu}}{q_{\nu}} k \mu \Psi \right\} \text{neutrinos}$$

massless or massive neutrinos $E_{
u}=\sqrt{p_{
u}^2+m_{
u}^2}$

Fourier: $\mathcal{N}(\vec{x}, \vec{p}, t) \longrightarrow \mathcal{N}(k, \mu, \eta)$

Expand in multipoles: $\mathcal{N}_{\ell}(k,\mu,\eta)$

CMB Power spectrum

$$C_{\ell} \propto \int dk [\ldots] \Theta_{\ell}(k)$$

Matter Power spect.

metric

$$P(k) \propto \langle \delta_{
m m}(k)^2 \rangle$$

$$k^{2} \left(\Phi + \Psi\right) = -32\pi G_{N} a^{2} \rho_{\rm r} \Theta_{\rm r,2}$$

$$\dot{\Theta} + ik\mu\Theta = -\dot{\Phi} - ik\mu\Psi - \dot{\tau} \left[\Theta_{0} - \Theta + \mu v_{b} - 1/2 \,\mathcal{P}_{2}(\mu)\Pi\right]
\dot{\tau} = d\tau/d\eta = -n_{e}\sigma_{T}a \qquad \Pi = \Theta_{2} + \Theta_{P2} + \Theta_{P0}
\dot{\Theta}_{P} + ik\mu\Theta_{P} = -\dot{\tau} \left[\Theta_{P} + 1/2\left(1 - \mathcal{P}_{2}(\mu)\right)\Pi\right]$$
photons

$$\left\{ \dot{\delta}_{
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ight\} {
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m b}+rac{\dot{a}}{a}v_{
m b}&=-ik\Psi+rac{\dot{ au}}{R}ig[v_{
m b}+3i\Theta_1ig] \end{aligned} } ext{ baryons}$$

$$\dot{\mathcal{N}}+i\frac{q_{\nu}}{E_{\nu}}k\mu\mathcal{N}=-\dot{\Phi}-i\frac{E_{\nu}}{q_{\nu}}k\mu\Psi\left\} \text{ neutrinos }$$

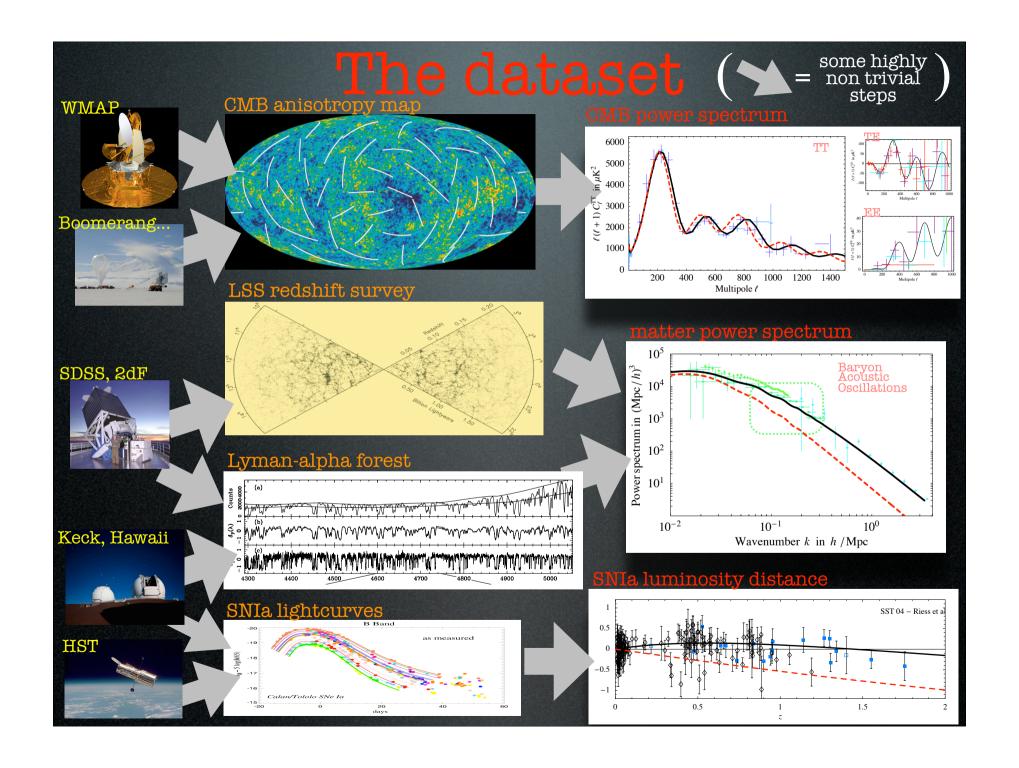
CMB Power spectrum

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$$k^{2}\left(\Phi + \Psi\right) = -32\pi G_{N}a^{2}\rho_{\mathrm{r}}\Theta_{\mathrm{r},2}$$
metric



CMB Temperature and Polarization:

- WMAP 3-years (TT, TE, EE spectra) wmap science Team, astro-ph/0603449 →
- Boomerang 2003 (TT, TE, EE) Boomerang Coll., astro-ph/0507494, astro-ph/0507507, astro-ph/0507514
- ACBAR (TT) Kuo et al., astro-ph/0212289 ->
- CAPMAP (EE) Barkats et al., astro-ph/0409380
- CBI (TT, EE) Readhead et al., astro-ph/0402359, astro-ph/0409569, Sievers at al., astro-ph/0509203 \rightarrow
- DASI (TE, EE) Leitch et al., astro-ph/0409357
- VSA (TT) Grainge et al., astro-ph/0212495

LSS galaxy redshift surveys: dealing with bias and non-linearities as

- SDSS SDSS Coll., astro-ph/0310725 →
- 2dF 2dF Coll., astro-ph/0501174

$$P_{\text{gal}}(k) = b^2 \frac{1 + Q k^2}{1 + A k} P(k)$$

Baryon Acoustic Oscillations: in terms of a

Eisenstein et al., astro-ph/0501171

in terms of a measurement of
$$A=\left(\frac{D_{\rm A}^2 cz}{H(z)}\right)^{1/3} \frac{\sqrt{\Omega_{\rm matter} H_0^2}}{0.35~c}$$

Lyman- α Forest:

- Croft Croft et al., astro-ph/0012324
- SDSS SDSS Coll., astro-ph/0407377

Type Ia Supernovae:

- SST Gold sample Riess et al., astro-ph/0402512 ->
- SNLS Astier et al., astro-ph/0510447 ->

Hubble constant:

$$h = 0.72 \pm 0.08$$

$$h = 0.72 \pm 0.08$$
 $H_0 = 100h \text{ km/sec/Mpc}$

The computational tool

We use our own code in MATHEMATICA5.2

as opposed to:

- evolve cosmological perturbations, CMBfast/CAMB

- compute spectra and

CMBfast/CAMB

- run statistical comparisons with data.

CosmoMC

(Recombination is implemented calling recfast.)

We adopt **gaussian** statistics.

For Standard Cosmology we obtain:

fit	A_s	h	n_s	au	$100\Omega_b h^2$	$\Omega_{ m DM} h^2$
WMAP3	0.80 ± 0.05	0.704 ± 0.033	0.935 ± 0.019	0.081 ± 0.030	2.24 ± 0.10	0.113 ± 0.010
Global	0.84 ± 0.04	0.729 ± 0.013	0.951 ± 0.012	0.121 ± 0.025	2.36 ± 0.07	0.117 ± 0.003

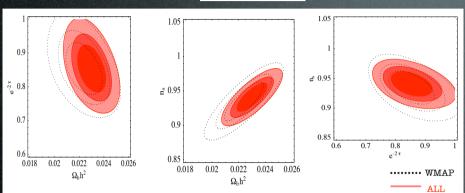
(assumes 3.04 massless, freely-streaming neutrinos).

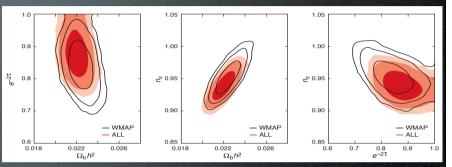
Comparing our code

Our analysis:



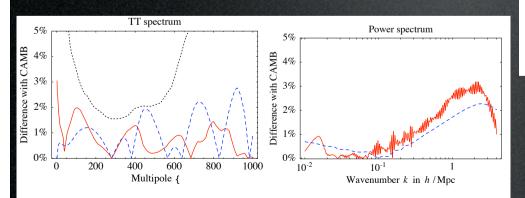
WMAP Science Team analysis:





[Spergel et al. WMAP 3yr results '05]

fit	A_s	h	n_s	au	$100\Omega_b h^2$	$\Omega_{\mathrm{DM}} h^2$
WMAP3	0.80 ± 0.05	0.704 ± 0.033	0.935 ± 0.019	0.081 ± 0.030	2.24 ± 0.10	0.113 ± 0.010
Global	0.84 ± 0.04	0.729 ± 0.013	0.951 ± 0.012	0.121 ± 0.025	2.36 ± 0.07	0.117 ± 0.003



	WMAP	WMAP+	WMAP+	WMAP +
	Only	SDSS	LRG	SN Gold
Parameter				
$100\Omega_b h^2$	$2.233^{+0.072}_{-0.091}$	$2.233^{+0.062}_{-0.086}$	$2.242^{+0.062}_{-0.084}$	$2.227^{+0.065}_{-0.082}$
$\Omega_m h^2$	$0.1268^{+0.0073}_{-0.0128}$	$0.1329^{+0.0057}_{-0.0109}$	$0.1337^{+0.0047}_{-0.0098}$	$0.1349^{+0.0054}_{-0.0106}$
h	$0.734^{+0.028}_{-0.038}$	$0.709^{+0.024}_{-0.032}$	$0.709^{+0.016}_{-0.023}$	$0.701^{+0.020}_{-0.026}$
A	$0.801^{+0.043}_{-0.054}$	$0.813^{+0.042}_{-0.052}$	$0.816^{+0.042}_{-0.049}$	$0.827^{+0.045}_{-0.053}$
au	$0.088^{+0.028}_{-0.034}$	$0.079^{+0.029}_{-0.032}$	$0.082^{+0.028}_{-0.033}$	$0.079^{+0.028}_{-0.034}$
n_s	$0.951^{+0.015}_{-0.019}$	$0.948^{+0.015}_{-0.018}$	$0.951^{+0.014}_{-0.018}$	$0.946^{+0.015}_{-0.019}$
σ_8	$0.744^{+0.050}_{-0.060}$	$0.772^{+0.036}_{-0.048}$	$0.781^{+0.032}_{-0.045}$	$0.784^{+0.035}_{-0.049}$
Ω_m	$0.238^{+0.027}_{-0.045}$	$0.266^{+0.025}_{-0.040}$	$0.267^{+0.017}_{-0.029}$	$0.276^{+0.022}_{-0.036}$

agreement is at **few** % level and within current precision of data

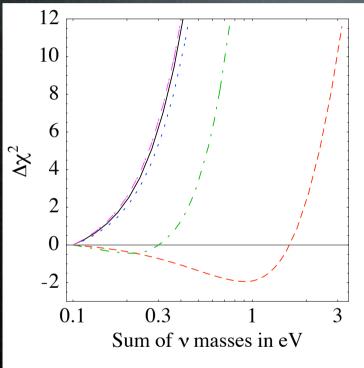
Results



How heavy are neutrinos?

Cosmology probes $\sum m_{\nu_i}$.





CMB only:

$$\sum m_{\nu_i} < 2.2 \text{ eV}$$
 (95% C.L.)

Global fit:

$$\sum m_{
u_i} < 0.40 \; {
m eV}$$
 (99.9% C.L.)

dropping Ly-alpha:

$$\sum m_{\nu_i} < 0.73 \, \text{eV}$$

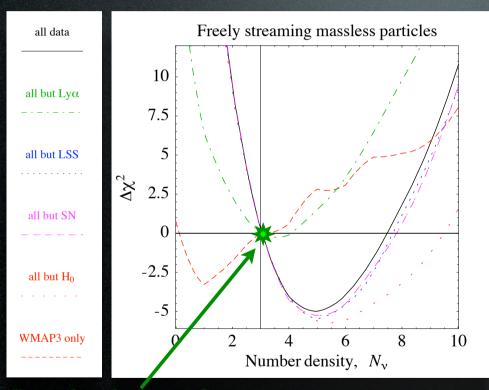
(99.9% C.L.)

Bottom Line: Cosmology gives dominant bound on $\sum m_{\nu_i}$; the bound tightens combining relatively less safe datasets.

Results New neutrinos?



All N_{ν} relativistic degrees of freedom contribute to the energy density.



Global fit:

$$N_{\nu} = 5 \pm 1$$

dropping Ly-alpha gives back

$$N_{\nu} \simeq 3$$

Standard cosmology

Bottom Line: Cosmology seems to suggest **5 neutrinos** (2 extra); but Ly-alpha are mainly driving the suggestion.

$$\dot{\Theta} + ik\mu\Theta = -\dot{\Phi} - ik\mu\Psi - \dot{\tau} \left[\Theta_0 - \Theta + \mu v_{\rm b} - 1/2 \,\mathcal{P}_2(\mu)\Pi\right] \\ \dot{\tau} = d\tau/d\eta = -n_e\sigma_T a \qquad \Pi = \Theta_2 + \Theta_{P2} + \Theta_{P0} \\ \dot{\Theta}_P + ik\mu\Theta_P = -\dot{\tau} \left[\Theta_P + 1/2 \left(1 - \mathcal{P}_2(\mu)\right)\Pi\right]$$
 photons

$$\left\{ \dot{\delta}_{
m dm} + ikv_{
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m b}&=-ik\Psi+rac{\dot{ au}}{R}ig[v_{
m b}+3i\Theta_1ig] \end{aligned} egin{aligned} ext{baryons} \end{aligned}$$

$$\dot{\mathcal{N}} + i \frac{q_{\nu}}{E_{\nu}} k \mu \mathcal{N} = -\dot{\Phi} - i \frac{E_{\nu}}{q_{\nu}} k \mu \Psi \right\} \text{neutrinos}$$

$$egin{aligned} \dot{\delta}_{\mathrm{x}} &= -(1+w)(3\dot{\Phi}+ikv_{\mathrm{x}}) \ \dot{v}_{\mathrm{x}} &= -ik\Psi + rac{\dot{a}}{a}\left(1-3w
ight)iv_{\mathrm{x}} - rac{w}{1+w}ik\delta_{\mathrm{x}} \end{aligned}
ight\} \, \mathrm{extra} \ \end{aligned}$$

$$k^{2}\Phi + 3\frac{\dot{a}}{a}\left(\dot{\Phi} - \Psi\frac{\dot{a}}{a}\right) = 4\pi G_{N}a^{2}\left[\rho_{m}\delta_{m} + 4\rho_{r}\delta_{r}\right]$$
$$k^{2}\left(\Phi + \Psi\right) = -32\pi G_{N}a^{2}\rho_{r}\Theta_{r,2}$$

Massive particles, interacting among themselves and with neutrinos (i.e.non freely streaming).



A fluid defined by $\delta_{\rm x}, v_{\rm x}$, with w=1/3 when rel, w=0 when NR.

Contribute to the Rel/NR energy densities.

metric

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