Neutrino properties from Cosmology: the usual and the less usual

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Neutrinos in the Cosmo

The Universe is made of: radiation, matter (DM+b+e), dark energy

Big Bang

3 min

400 million y

1-10 billion y

CMB

BBN

RD

R\text{Meq}

\text{redshift} \ 1+z

\Omega

0

1

0.2

0.4

0.6

0.8

1

M

\Lambda

\text{equ}

\text{Today}

\Omega_\text{RD}

\Omega_\text{MD}

\Omega_\text{\Lambda\text{equ}}
Neutrinos in the Cosmo

The Universe is made of: radiation, matter (DM+b+e), dark energy and neutrinos

Neutrinos are significant because:
- main component of the rel energy density that sets expansion rate of the Universe
- (ordinary neutrinos have a mass, so) turn from Rel to NRel at a crucial time
- may free-stream or interact among themselves, or with new light particles
Neutrinos in the Cosmo

So what “neutrinos”?

- 3 ordinary, SM neutrinos
- Extra light degrees of freedom, very weakly coupled to SM forces

So what properties are probed by cosmology?

- Neutrino number
- Total neutrino mass
- Non-conventional interactions

What are the relevant cosmological probes?

- BBN \( (T \sim \text{MeV}, \text{flavor is important, primordial plasma}) \)
- Later cosmology i.e. CMB+LSS \( (T \lesssim \text{eV}, \approx m_\nu, \text{gravity is the only force}) \)

Cosmological data are (mostly) not sensitive to:

\[ \theta_{\text{active}}, m_{1,2,3} \text{ (or } \Delta m^2_{\text{active}}), \text{ CP-violation...} \]
Neutrinos in the Cosmo

So what “neutrinos”?  

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- BBN ($T \sim$ MeV, flavor is important, primordial plasma)  
- Later cosmology i.e. CMB+LSS ($T \lesssim$ eV, $\approx m_{\nu}$, gravity is the only force)

Cosmological data are (mostly) not sensitive to:  

$\theta_{\text{active}}$, $m_{1,2,3}$ (or $\Delta m_{\text{active}}^2$), $CP$–violation...
Neutrinos in CMB+LSS

Neutrinos affect (indirectly, i.e. gravitationally) the evolution of cosmological perturbations in radiation and matter.

\[ N_\nu, \sum m_\nu \]

... 

\[ \Omega_b, \Omega_{DM}, \tau, A_s, H_0, n_s \]

WMAP

CMB

LSS

2dF, SDSS, Ly-A
Formalism
(=cosmological perturbation theory in one slide)

\[ \dot{\Theta} + ik\mu\Theta = -\dot{\Phi} - ik\mu\Psi - \dot{\tau} \left[ \Theta_0 - \Theta + \mu\nu_b - 1/2 \mathcal{P}_2(\mu)\Pi \right] \]
\[ \dot{\Pi} = \Theta_2 + \Theta_{P2} + \Theta_{P0} \]

\[ \dot{\Theta}_P + ik\mu\Theta_P = -\dot{\tau} \left[ \Theta_P + 1/2(1 - \mathcal{P}_2(\mu))\Pi \right] \]

\[ \dot{\delta}_{dm} + ik\nu_{dm} = -3\dot{\Phi} \]
\[ \dot{\nu}_{dm} + \frac{\dot{a}}{a} \nu_{dm} = -ik\Psi \]

\[ \dot{\delta}_b + ik\nu_b = -3\dot{\Phi} \]
\[ \dot{\nu}_b + \frac{\dot{a}}{a} \nu_b = -ik\Psi + \frac{\dot{\tau}}{R} \left[ \nu_b + 3i\Theta_1 \right] \]

\[ \dot{\delta}_{dm} + ik\nu_{dm} = -3\dot{\Phi} + i\frac{E_\nu}{q_\nu} k\mu \dot{\nu} \]

\[ k^2 \Phi + 3 \frac{\dot{a}}{a} \left( \dot{\Phi} - \Psi \frac{\dot{a}}{a} \right) = 4\pi G_N a^2 \left[ \rho_m \delta_m + 4\rho_r \delta_r \right] \]

\[ k^2 (\Phi + \Psi) = -32\pi G_N a^2 \rho_r \Theta_{r,2} \]
Formalism

(=cosmological perturbation theory in one slide)

\[ \dot{\Theta} + i k \mu \Theta = -\dot{\Phi} - i k \mu \Psi - \dot{\tau} \left[ \Theta_0 - \Theta + \mu v_b - 1/2 \mathcal{P}_2(\mu) \Pi \right] \]

\[ \dot{\tau} = d\tau/d\eta = -n_e \sigma T a \quad \Pi = \Theta_2 + \Theta P_2 + \Theta P_0 \]

\[ \Theta = \frac{\delta T}{T} \quad f(\vec{x}, \vec{p}) = \frac{1}{e^{T + \delta T} - 1} \]

Fourier: \( \Theta(\vec{x}, \vec{p}, t) \rightarrow \Theta(k, \mu, \eta) \quad \mu = \hat{k} \cdot \hat{p} \)

Expand in multipoles:

\[ \Theta_\ell(k, \eta) = \frac{1}{(-1)^\ell} \int_{-1}^1 d\mu \frac{1}{2} \mathcal{P}(\mu) \Theta(k, \mu, \eta) \]

\[ \dot{v}_b + \frac{\dot{a}}{a} v_b = -i k \Psi + \frac{\dot{\tau}}{R} [v_b + 3i \Theta_1] \]

\[ \dot{N} + i \frac{q_\nu}{E_\nu} k \mu N = -\dot{\Phi} - i \frac{E_\nu}{q_\nu} k \mu \Psi \]

baryons

\[ k^2 \dot{\Phi} + 3 \frac{a}{\dot{a}} \left( \dot{\Phi} - \Psi \frac{\dot{a}}{a} \right) = 4\pi G_N a^2 \left[ \rho_m \delta_m + 4 \rho_r \delta_r \right] \]

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neutrinos

photons

CMB Power spectrum

\[ C_\ell \propto \int dk \ldots \Theta_\ell(k) \]

Matter Power spect.

\[ P(k) \propto \langle \delta_m(k)^2 \rangle \]
Formalism
 (=cosmological perturbation theory in one slide)

\[ \dot{\Theta} + ik\mu\Theta = -\dot{\Phi} - ik\mu\Psi - \dot{\tau}\left[\Theta_0 - \Theta + \mu v_b - 1/2 P_2(\mu)\Pi\right] \]

\[ \dot{\Theta}_P + ik\mu\Theta_P = -\dot{\tau}\left[\Theta_P + 1/2(1 - P_2(\mu))\Pi\right] \]

\[ \dot{\delta}_{dm} + ikv_{dm} = -3\dot{\Phi} \]

\[ \dot{v}_{dm} + \frac{\dot{a}}{a}v_{dm} = -ik\Psi \]

\[ \delta_{dm} = \frac{\delta\rho_{dm}}{\rho_{dm}} \]

\[ \rho_{dm}(\vec{x}, t) = \rho_{dm}^0 \left(1 + \delta_{dm}(\vec{x}, t)\right) \]

and velocity \( v_{dm} \)

Fourier:

\[ \delta_{dm}(\vec{x}, t) \rightarrow \delta_{dm}(k, \eta) \]

\[ v_{dm}(\vec{x}, t) \rightarrow v_{dm}(k, \eta) \]

\[ k^2 \Phi + 3 \frac{\dot{a}}{a} \left(\dot{\Phi} - \Psi \frac{\dot{a}}{a}\right) = 4\pi G_N a^2 \left[\rho_m \delta_m + 4\rho_r \delta_r\right] \]

\[ k^2 (\Phi + \Psi) = -32\pi G_N a^2 \rho_r \Theta_{r,2} \]

\[ C_{\ell} \propto \int dk \left[ \ldots \right] \Theta_{\ell}(k) \]

\[ P(k) \propto \langle \delta_m(k)^2 \rangle \]

\[ \text{CMB Power spectrum} \]

\[ \text{Matter Power spect.} \]
Formalism
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\[ \dot{\Theta} + i k \mu \Theta = -\dot{\Phi} - i k \mu \Psi - \dot{\tau} \left[ \Theta_0 - \Theta + \mu v_b - 1/2 \mathcal{P}_2(\mu) \Pi \right] \]
\[ \dot{\Theta}_P + i k \mu \Theta_P = -\dot{\tau} \left[ \Theta_P + 1/2 \left( 1 - \mathcal{P}_2(\mu) \right) \Pi \right] \]

\[ \dot{\delta}_{dm} + i k v_{dm} = -3 \dot{\Phi} \quad \text{dark matter} \]
\[ \dot{v}_{dm} + \frac{\dot{a}}{a} v_{dm} = -i k \Psi \]

\[ \dot{\delta}_b + i k v_b = -3 \dot{\Phi} \quad \text{baryons} \]
\[ \dot{v}_b + \frac{\dot{a}}{a} v_b = -i k \Psi + \frac{\dot{\tau}}{R} \left[ v_b + 3 i \Theta_1 \right] \]

\[ \delta_b(k, \eta) \quad \text{Thomson scattering} \quad e^- \gamma \leftrightarrow e^- \gamma \]

\[ k^2 \Phi + 3 \frac{\dot{a}}{a} \left( \dot{\Phi} - \Psi \frac{\dot{a}}{a} \right) = 4\pi G_N a^2 \left[ \rho_m \delta_m + 4 \rho_r \delta_r \right] \quad \text{metric} \]

\[ k^2 (\Phi + \Psi) = -32 \pi G_N a^2 \rho_r \Theta_r,2 \]

\[ \Pi = \Theta_0 + \Theta_P + \Theta_{P_0} \]

\[ \dot{\tau} = d\tau/d\eta = -n_e \sigma_T a \]

\[ C_\ell \propto \int dk [\ldots] \Theta_\ell(k) \]

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\[ \dot{\Theta} + i k \mu \Theta = -\dot{\Phi} - i k \mu \Psi - \dot{\tau} [\Theta_0 - \Theta + \mu \nu_b - 1/2 \mathcal{P}_2(\mu) \Pi] \]
\[ \dot{\Theta}_P + i k \mu \Theta_P = -\dot{\tau} [\Theta_P + 1/2 (1 - \mathcal{P}_2(\mu)) \Pi] \]
\[ \delta_{dm} + i k \nu_{dm} = -3 \dot{\Phi} \]
\[ \dot{\nu}_{dm} + \frac{\dot{a}}{a} \nu_{dm} = -i k \Psi \]

scalar metric perturbations:
\[ g_{\mu \nu} = \eta_{\mu \nu} + \delta \eta_{\mu \nu}(\Psi, \Phi) \]
\[ g_{\mu \nu} = \begin{pmatrix} -1 - 2\Psi & 0 & 0 & 0 \\ 0 & a^2(1 + 2\Phi) & 0 & 0 \\ 0 & 0 & a^2(1 + 2\Phi) & 0 \\ 0 & 0 & 0 & a^2(1 + 2\Phi) \end{pmatrix} \]

Fourier:
\[ \Psi(\vec{x}, t) \longrightarrow \Psi(k, \eta) \]
\[ \Phi(\vec{x}, t) \longrightarrow \Phi(k, \eta) \]

\[ k^2 \Phi + 3 \frac{\dot{a}}{a} \left( \dot{\Phi} - \Psi \frac{\dot{a}}{a} \right) = 4\pi G_N a^2 \left[ \rho_m \delta_m + 4 \rho_\gamma \delta_r \right] \]
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CMB Power spectrum
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Matter Power spect.
\[ P(k) \propto \langle \delta_m(k)^2 \rangle \]
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\[
\dot{\Theta} + ik\mu \Theta = -\dot{\Phi} - ik\mu \Psi - \tau [\Theta_0 - \Theta + \mu \nu_b - 1/2 P_2(\mu) \Pi]
\]

\[
\dot{\Theta}_P + ik\mu \Theta_P = -\dot{\tau} [\Theta_P + 1/2 (1 - P_2(\mu)) \Pi]
\]

photons

\[
\dot{\delta}_{dm} + ik\nu_{dm} = -3\dot{\Phi} \quad \text{dark matter}
\]

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\dot{\nu}_{dm} + \frac{\dot{a}}{a} \nu_{dm} = -ik\Psi
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\dot{\delta}_b + ik\nu_{b} = -3\dot{\Phi} \quad \text{baryons}
\]

\[
\dot{\nu}_b + \frac{\dot{a}}{a} \nu_b = -ik\Psi + \frac{\dot{\tau}}{R} [\nu_b + 3i\Theta_1]
\]

\[
\dot{N} + i \frac{q_\nu}{E_\nu} k\mu N = -\dot{\Phi} - i \frac{E_\nu}{q_\nu} k\mu \Psi \quad \text{neutrinos}
\]

neutrinos

massless or massive neutrinos \quad E_\nu = \sqrt{p_\nu^2 + m_\nu^2}

Fourier: \( N(\vec{x}, \vec{p}, t) \rightarrow N(k, \mu, \eta) \)

Expand in multipoles: \( N_\ell(k, \mu, \eta) \)

CMB Power spectrum

\[ C_\ell \propto \int dk [\ldots] \Theta_\ell(k) \]

Matter Power spect.

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\[ k^2 \Phi + 3 \frac{\dot{a}}{a} \left( \dot{\Phi} - \Psi \frac{\dot{a}}{a} \right) = 4\pi G_N a^2 \left[ \rho_m \delta_m + 4\rho_r \delta_r \right] \quad \text{metric} \]

\[ k^2 (\Phi + \Psi) = -32\pi G_N a^2 \rho_r \Theta_{r,2} \]
### An application:

**the effect of neutrino mass on the Matter Power Spectrum**

- let’s follow $\delta_{dm}$ during MD (matter perturbations don’t grow during RD)

\[
\begin{align*}
\dot{\delta}_{dm} + ikv_{dm} &= -3\dot{\Phi} \\
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\end{align*}
\]

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k^2\Phi + 3\frac{\dot{a}}{a}\left(\dot{\Phi} - \Psi\frac{\dot{a}}{a}\right) &= 4\pi G_N a^2 [\rho_m \delta_m + 4\rho_r \delta_r] \\
k^2 (\Phi + \Psi) &= -32\pi G_N a^2 \rho_r \Theta_{r,2}
\end{align*}
\]

**Matter Power spect.**

\[
P(k) \propto \langle \delta_m(k)^2 \rangle
\]
An application: the effect of neutrino mass on the Matter Power Spectrum

- let's follow $\delta_{\text{dm}}$ during MD (matter perturbations don't grow during RD)

\[ \ddot{\delta}_{\text{dm}} + \frac{\dot{a}}{a} \dot{\delta}_{\text{dm}} \simeq k^2 \Phi \]

\[
\begin{align*}
\dot{\delta}_{\text{dm}} + i k v_{\text{dm}} &= -3 \dot{\Phi} \\
\dot{v}_{\text{dm}} + \frac{\dot{a}}{a} v_{\text{dm}} &= -i k \Psi
\end{align*}
\]

dark matter

\[ k^2 \Phi = 4 \pi G_N a^2 [\rho_m \delta_m + 4 \rho_r \Theta_{r,0} + \frac{3 H a}{K} (i \rho_m v_m + 4 \rho_r \Theta_{r,1})] \]

\[
\begin{align*}
 k^2 (\Phi + \Psi) &= -32 \pi G_N a^2 \rho_r \Theta_{r,2} \\
 k^2 \Phi + 3 \frac{\dot{a}}{a} \left( \dot{\Phi} - \Psi \frac{\dot{a}}{a} \right) &= 4 \pi G_N a^2 [\rho_m \delta_m + 4 \rho_r \delta_r]
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metric

Matter Power spect.

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An application: the effect of neutrino mass on the Matter Power Spectrum

- let’s follow $\delta_{dm}$ during MD (matter perturbations don’t grow during RD)

\[ \ddot{\delta}_{dm} + \frac{\dot{a}}{a} \dot{\delta}_{dm} \simeq 4\pi G_N a^2 \rho_m \delta_m \]  

(Newton eq. for $\delta_{dm}$)  

($\dot{\cdot} = \frac{d}{d\eta}$)
An application:
the effect of neutrino mass on the Matter Power Spectrum

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  \[ \ddot{\delta}_{dm} + \frac{\dot{a}}{a} \dot{\delta}_{dm} \simeq 4\pi G_N a^2 \rho_m \delta_m \]  
  (Newton eq. for $\delta_{dm}$)  
  \( \cdot = \frac{d}{d\eta} \)

- with massless neutrinos:

  FRW eq.  
  \[ H^2 = \frac{8}{3} \pi G_N \rho_m \]  
  with  
  \( \rho_m = (\rho_{dm} + \rho_b) \propto a^{-3} \Rightarrow a \propto t^{2/3} \)

  so  
  \[ \delta''_{dm} + \frac{4}{3} \frac{1}{t} \delta'_{dm} - \frac{2}{3} \frac{1}{t^2} \delta_{dm} = 0 \Rightarrow \text{growing solution} \quad \delta_{dm} \propto t^{2/3} \propto a \]
An application: the effect of neutrino mass on the Matter Power Spectrum

- let’s follow $\delta_{dm}$ during MD (matter perturbations don’t grow during RD)

$$\ddot{\delta}_{dm} + \frac{\dot{a}}{a} \dot{\delta}_{dm} \simeq 4\pi G_N a^2 \rho_m \delta_m$$

(Newton eq. for $\delta_{dm}$) \quad (\dot{\cdot} = \frac{d}{d\eta})

- with massless neutrinos:

  FRW eq. $H^2 = \frac{8}{3} \pi G_N \rho_m$ with $\rho_m = (\rho_{dm} + \rho_b) \propto a^{-3} \Rightarrow a \propto t^{2/3}$

  so $\delta_{dm}'' + \frac{4}{3} \frac{1}{t} \delta_{dm}' - \frac{2}{3} \frac{1}{t^2} \delta_{dm} = 0 \Rightarrow$ growing solution $\delta_{dm} \propto t^{2/3} \propto a$

- with massive neutrinos:

  FRW eq. with $\rho_m = (\rho_{dm} + \rho_b + \rho_\nu) = \rho_m (1 + f_\nu)$, $f_\nu = \frac{\Omega_\nu}{\Omega_m} \Rightarrow a \propto (1 + f_\nu)^{1/3} t^{2/3}$

  $$\Omega_\nu = \sum m_{\nu_i} / 93 \text{ eV}$$

but $\delta_\nu = 0$ because neutrinos don’t cluster (at $k \gg k_{NR}$, see below)

i.e. massive neutrinos contribute to the energy density of the Universe during MD but they don’t source in the Newton equation for $\delta_{dm}$!

thus $\delta_{dm}'' + \frac{4}{3} \frac{1}{t} \delta_{dm}' - \frac{2}{3} (1 - f_\nu) \frac{1}{t^2} \delta_{dm} = 0 \Rightarrow \delta_{dm} \propto t^{-1 + \frac{\sqrt{25 - 24 f_\nu}}{6}} \propto a^{1 - \frac{3}{5} f_\nu}$. 

\[ \delta_{dm} \]
An application: 

the effect of neutrino mass on the Matter Power Spectrum

- let’s follow $\delta_{dm}$ during MD (matter perturbations don’t grow during RD)

\[ \ddot{\delta}_{dm} + \frac{\dot{a}}{a} \dot{\delta}_{dm} \simeq 4\pi G_N a^2 \rho_m \delta_m \quad \text{(Newton eq. for $\delta_{dm}$)} \quad \left( \dot{\cdot} = \frac{d}{d\eta} \right) \]

- with massless neutrinos: 

FRW eq. $H^2 = \frac{8}{3} \pi G_N \rho_m$ with $\rho_m = (\rho_{dm} + \rho_b) \propto a^{-3} \Rightarrow a \propto t^{2/3}$

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thus $\dddot{\delta}_{dm} + \frac{4}{3} \frac{1}{t} \ddot{\delta}_{dm} - \frac{2}{3} \left(1 - f_\nu\right) \frac{1}{t^2} \delta_{dm} = 0 \Rightarrow \delta_{dm} \propto t^{-1+\sqrt{25-24f_\nu}/6} \propto a^{1-3f_\nu/5}$

- so effective suppression of the growth is: $\frac{\delta_{dm}^{(m_\nu\neq0)}}{\delta_{dm}^{(m_\nu=0)}} \simeq (a_{NR})^{\frac{3}{5}f_\nu}$

or in terms of $P(k) \propto \langle \delta_{dm} \rangle^2$: 

\[ \frac{P^{(m_\nu\neq0)}(k)}{P^{(m_\nu=0)}(k)} \simeq (a_{NR})^{\frac{6}{5}f_\nu} \quad \text{modeled by} \quad \frac{\Delta P}{P} \simeq -8f_\nu \]

(a_{NR} because at that point massless and massive coincide)
An application: the effect of neutrino mass on the Matter Power Spectrum

- at what scales is the effect relevant?

The free streaming scale is the distance traveled by a neutrino in a Hubble time

$$\lambda_{FS}(t) \simeq \frac{v(t)}{H(t)}$$

For massive neutrinos

$$v(t) \simeq \frac{3T_{\nu}}{m_{\nu}} \propto \frac{1}{a} \left( \frac{1 \text{ eV}}{m_{\nu}} \right) \text{ km/sec}$$

so

$$\lambda_{FS} \propto \sqrt{\Omega_m} a^{-3} \left( \frac{1 \text{ eV}}{m_{\nu}} \right) \text{ km/sec} \propto a^{1/2}$$

But the size of the Universe scales as

$$\frac{1}{H(t)} \propto a^{3/2}$$

so \( \lambda_{FS} \) starts lagging behind \( H^{-1} \) when \( \nu \) become NR, at

$$a_{NR}^{-1} \simeq 2.1 \times 10^3 \frac{m_{\nu}}{\text{eV}}$$

(i.e. \( m_{\nu} \) becomes relevant)

The effect of free streaming affects comoving scales inside

$$\frac{\lambda_{NR}}{a} = \frac{\lambda_{FS}}{a_{NR}}$$

or in terms of

$$k_{NR} = 2\pi \left( \frac{\lambda_{NR}}{a} \right)^{-1}$$

at

$$k \gg k_{NR} = 0.018 \Omega_m^{-1/2} \left( \frac{m_{\nu}}{\text{eV}} \right)^{1/2} h_0 \text{ Mpc}^{-1}$$
An application:
the effect of neutrino mass on the Matter Power Spectrum

In summary

\[ \Delta P \approx -8 f_\nu \]

Caveat: plots for illustrative purposes only, all parameters except \( m_\nu \) are held fixed.

\[ k_{NR} = 0.018 \Omega_m^{-1/2} \left( \frac{m_\nu}{\text{eV}} \right)^{1/2} h_0 \text{Mpc}^{-1} \]

\[ \frac{\Delta P}{P} \approx -8 f_\nu \quad \left( f_\nu = \frac{\sum m_{\nu_i}}{\Omega_m / 93 \text{eV}} \right) \]
We have the formalism to compute the effect on cosmological observables.

Let’s compare quantitatively with cosmological data.
The dataset

- **WMAP**: CMB anisotropy map
- **Boomerang**: CMB power spectrum
- **SDSS, 2dF**: LSS redshift survey
- **Keck, Hawaii**: Lyman-alpha forest
- **HST**: SNIa lightcurves
- **SNIa luminosity distance**: Matter power spectrum

(→ = some highly non-trivial steps)
The dataset

**CMB Temperature and Polarization:**
- **WMAP 3-years** (TT, TE, EE spectra) WMAP Science Team, astro-ph/0603449 →
- ACBAR (TT) Kuo et al., astro-ph/0212289 →
- CAPMAP (EE) Barkats et al., astro-ph/0409380
- DASI (TE, EE) Leitch et al., astro-ph/0409367
- VSA (TT) Grainge et al., astro-ph/0212495

**LSS galaxy redshift surveys:** dealing with bias and non-linearities as
- SDSS SDSS Coll., astro-ph/0310725 →
- 2dF 2dF Coll., astro-ph/0501174

**Baryon Acoustic Oscillations:** in terms of a measurement of
- Eisenstein et al., astro-ph/0501171

**Lyman-α Forest:**
- Croft Croft et al., astro-ph/0012324
- SDSS SDSS Coll., astro-ph/0407377

**Type Ia Supernovae:**
- SST Gold sample Riess et al., astro-ph/0402512 →
- SNLS Astier et al., astro-ph/0510447 →

**Hubble constant:**
- h = 0.72 ± 0.08  \( H_0 = 100h \) km/sec/Mpc
The computational tool

We use our own code in *Mathematica* to
- evolve cosmological perturbations,
- compute spectra and
- run statistical comparisons with data.

As opposed to:
- CMBfast/CAMB
- CosmoMC

Line-of-sight approach, Newtonian gauge.
Recombination is implemented calling `recfast`.
SZ background is marginalized over.

We adopt *gaussian* statistics.
The computational tool

We use **our own code** in **Mathematica** to
- evolve cosmological perturbations,
- compute spectra and
- run statistical comparisons with data.

Line-of-sight approach, Newtonian gauge.
Recombination is implemented calling `recfast`.
SZ background is marginalized over.

We adopt **gaussian** statistics.

- slower, not fully optimized,
  intrinsic gaussian “systematics”
- customizable, analytic computations,
  analytic dependence on cosmo parameters

as opposed to:

CMBfast/CAMB

CosmoMC

MCMC
Comparing our code

Our analysis: WMAP Science Team analysis:

![Image of analysis results]

### Table: Parameter Comparison

<table>
<thead>
<tr>
<th>Parameter</th>
<th>WMAP Only</th>
<th>WMAP + SDSS</th>
<th>WMAP + LRG</th>
<th>WMAP + SN Gold</th>
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agreement is at few % level and within current precision of data
Results

How heavy are neutrinos?

(3 massive neutrinos)
Results

How heavy are neutrinos?

Cosmology probes $\sum m_{\nu_i}$.

Caveat: plots for illustrative purposes only, all parameters except $m_\nu$ are held fixed.

$$k_{NR} = 0.018 \Omega_m^{-1/2} \left( \frac{m_\nu}{\text{eV}} \right)^{1/2} h_0 \text{ Mpc}^{-1}$$

$$\frac{\Delta P}{P} \simeq -8 f_\nu \quad \left( f_\nu = \frac{\sum m_{\nu_i} / 93 \text{ eV}}{\Omega_m} \right)$$
Results

How heavy are neutrinos?

Cosmology probes $\sum m_{\nu_i}$.

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$$k_{NR} = 0.018 \Omega_m^{-1/2} \left( \frac{m_{\nu}}{\text{eV}} \right)^{1/2} h_0 \text{Mpc}^{-1}$$

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Results

How heavy are neutrinos?

Cosmology probes $\sum m_{\nu_i}$.

![Graph showing constraints on the sum of neutrino masses.](image)

**CMB only:**

$$\sum m_{\nu_i} < 2.2 \text{ eV}$$  
(95% C.L.)

**Global fit:**

$$\sum m_{\nu_i} < 0.40 \text{ eV}$$  
(99.9% C.L.)

**dropping Ly-alpha:**

$$\sum m_{\nu_i} < 0.73 \text{ eV}$$  
(99.9% C.L.)

**Bottom Line:** Cosmology gives dominant bound on $\sum m_{\nu_i}$; the bound tightens combining relatively less safe datasets.
Results
New neutrinos?
Results
New neutrinos?

\[ \dot{\Theta} + i k \mu \Theta = -\dot{\Phi} - i k \mu \Psi - \dot{\tau} \left[ \Theta_0 - \Theta + \mu v_b - 1/2 \mathcal{P}_2(\mu) \Pi \right] \]

\[ \dot{\Theta}_P + i k \mu \Theta_P = -\dot{\tau} \left[ \Theta_P + 1/2 \left( 1 - \mathcal{P}_2(\mu) \right) \Pi \right] \]

\[ \dot{\delta}_{dm} + i k v_{dm} = -3 \dot{\Phi} \]

\[ \dot{\nu}_{dm} + \frac{\dot{a}}{a} \nu_{dm} = -i k \Psi \]

\[ \dot{\delta}_b + i k v_b = -3 \dot{\Phi} \]

\[ \dot{\nu}_b + \frac{\dot{a}}{a} \nu_b = -i k \Psi + \frac{\dot{\tau}}{R} \left[ \nu_b + 3i \Theta_1 \right] \]

\[ \dot{N} + i \frac{q_\nu}{E_\nu} k \mu N = -\dot{\Phi} - i \frac{E_\nu}{q_\nu} k \mu \Psi \]

\[ k^2 \Phi + 3 \frac{\dot{a}}{a} \left( \dot{\Phi} - \Psi \frac{\dot{a}}{a} \right) = 4\pi G_N a^2 \left[ \rho_m \delta_m + 4\rho_r \delta_r \right] \]

\[ k^2 (\Phi + \Psi) = -32\pi G_N a^2 \rho_r \Theta_r,2 \]
Results

New neutrinos?

\[ \dot{\Theta} + ik\mu \Theta = -\Phi - ik\mu \Psi - \dot{\tau} [\Theta_0 - \Theta + \mu v_b - 1/2 P_2(\mu)\Pi] \]

\[ \dot{\Theta}_P + ik\mu \Theta_P = -\dot{\tau} [\Theta_P + 1/2 (1 - P_2(\mu))\Pi] \]

\[ \dot{\delta}_{dm} + ik v_{dm} = -3\dot{\Phi} \quad \{\text{dark matter}\} \]

\[ \dot{v}_{dm} + \frac{\dot{a}}{a} v_{dm} = -ik \Psi \]

\[ \dot{\delta}_b + ik v_b = -3\dot{\Phi} \quad \{\text{baryons}\} \]

\[ \dot{v}_b + \frac{\dot{a}}{a} v_b = -ik \Psi + \frac{\dot{\tau}}{R} [v_b + 3i\Theta_1] \]

\[ \dot{N} + i \frac{q_\nu}{E_\nu} k\mu N = -\dot{\Phi} - i \frac{E_\nu}{q_\nu} k\mu \Psi \quad \{\text{neutrinos}\} \]

\[ k^2 \Phi + 3 \frac{\dot{a}}{a} \left( \dot{\Phi} - \Psi \frac{\dot{a}}{a} \right) = 4\pi G_N a^2 \left[ \rho_m \delta_m + 4\rho_r \delta_r \right] \quad \{\text{metric}\} \]

\[ k^2 (\Phi + \Psi) = -32\pi G_N a^2 \rho_r \Theta_{r,2} \]

\[
\begin{align*}
\dot{\tau} &= d\tau/d\eta = -n_e \sigma_T a \\
\Pi &= \Theta_2 + \Theta_P + \Theta_{P0} \quad \text{photons} \\
\text{CMB Power spectrum} &\quad G_\ell \propto \langle \delta_m(k)^2 \rangle \\
\end{align*}
\]

\[ N_\nu \text{ rel degrees of freedom contribute to the energy density} \]

\[ \rho_r = \rho_\gamma \left[ 1 + \frac{7}{8} N_\nu \left( \frac{T_\nu}{T} \right)^4 \right] \]

\[ \text{with} \quad \frac{T_\nu}{T} = \left( \frac{4}{11} \right)^{1/3} \]
Results
New neutrinos?

Caveat: plots for illustrative purposes only, all parameters except $N_\nu$ are held fixed (here this caveat is particularly important).

\[ N_\nu = 3 \quad \text{and} \quad N_\nu = 5 \]
Results
New neutrinos?

All $N_{\nu}$ relativistic degrees of freedom contribute to the energy density.

Global fit:

$N_{\nu} = 5 \pm 1$

dropping Ly-alpha gives back

$N_{\nu} \approx 3$

Standard cosmology

Bottom Line: Cosmology seems to suggest 5 neutrinos (2 extra); but Ly-alpha are mainly driving the suggestion.
All $N_\nu$ relativistic degrees of freedom contribute to the energy density.

Standard cosmology

Bottom Line: Cosmology seems to suggest 5 neutrinos (2 extra); but Ly-alpha are mainly driving the suggestion.
Results
New sticky particles?

3.04 $\Delta N_\nu$
\[ \dot{\Theta} + i k \mu \Theta = -\Phi - i k \mu \Psi - \tau \left[ \Theta_0 - \Theta + \mu v_b - 1/2 P_2(\mu) \Pi \right] \]

\[ \dot{\Theta}_P + i k \mu \Theta_P = -\tau \left[ \Theta_P + 1/2 (1 - P_2(\mu)) \Pi \right] \]

\[ \dot{\delta}_{dm} + i k v_{dm} = -3 \Phi \]

\[ \dot{v}_{dm} + \frac{\dot{a}}{a} v_{dm} = -i k \Psi \]

\[ \dot{\delta}_b + i k v_b = -3 \Phi \]

\[ \dot{v}_b + \frac{\dot{a}}{a} v_b = -i k \Psi + \frac{\dot{\tau}}{R} [v_b + 3i \Theta_1] \]

\[ \dot{N} + i \frac{q_{\nu}}{E_{\nu}} k \mu N = -\Phi - i \frac{E_{\nu}}{q_{\nu}} k \mu \Psi \]

\[ \dot{\delta}_x + i\frac{4}{3} k v_x = -4 \Phi \]

\[ \dot{v}_x + \frac{i}{4} k \delta_x = -i k \Psi \]

\[ k^2 \Phi + 3 \frac{\dot{a}}{a} \left( \dot{\Phi} - \Psi - \dot{\Theta} \right) = 4\pi G_N a^2 \left[ \rho_m \delta_m + 4 \rho_r \delta_r \right] \]

\[ k^2 (\Phi + \Psi) = -32\pi G_N a^2 \rho_r \Theta_{r,2} \]

\[ \dot{\tau} = d\tau/d\eta = -n_e \sigma_T a \]

Massless particles, interacting among themselves (i.e. non freely streaming) at the time of CMB formation.

- e.g. a scalar \( \varphi \) with \( \lambda \varphi^4 \)
- e.g. scalar + fermion with \( \lambda' \varphi \nu_s^2 \)
- e.g. fermions with \( \langle N N \rangle \) ...

\[ \nabla \cdot (\delta_x, v_x) \]

A relativistic fluid: \( \delta_x, v_x \).

Contributes \( \Delta N_{\nu} \cdot \delta_x \) to the rel energy density.
ΔN_ν extra massless particles interacting among themselves.

Bottom Line: Cosmology constrains extra massless sticky particles.
Results
New massive neutrinos?
\[ \Delta N_{\nu}, m_s \]
\[\dot{\Theta} + i k \mu \Theta = - \dot{\Phi} - i k \mu \Psi - \dot{\tau} \left[\Theta_0 - \Theta + \mu \nu_b - 1/2 \mathcal{P}_2(\mu) \Pi\right]\]

\[\dot{\Theta}_P + i k \mu \Theta_P = - \dot{\tau} \left[\Theta_P + 1/2 (1 - \mathcal{P}_2(\mu)) \Pi\right]\]

\[\dot{\delta}_m + i k \nu_{dm} = -3 \dot{\Phi}\]

\[\dot{\nu}_{dm} + \frac{\dot{a}}{a} \nu_{dm} = - i k \Psi\]

\[\dot{\delta}_b + i k \nu_b = -3 \dot{\Phi}\]

\[\dot{\nu}_b + \frac{\dot{a}}{a} \nu_b = - i k \Psi + \frac{\dot{\tau}}{R} \left[\nu_b + 3 i \Theta_1\right]\]

\[\dot{N} + i \frac{q_{\nu}}{E_{\nu}} k \mu N = - \dot{\Phi} - i \frac{E_{\nu}}{q_{\nu}} k \mu \Psi\]

\[k^2 \Phi + 3 \frac{\dot{a}}{a} \left(\dot{\Phi} - \Psi \frac{\dot{a}}{a}\right) = 4\pi G_N a^2 \left[\rho_m \delta_m + 4 \rho_r \delta_r\right]\]

\[k^2 (\Phi + \Psi) = - 32\pi G_N a^2 \rho_r \Theta_{r,2}\]

Results

New massive neutrinos?

\[\Delta N_{\nu}, m_s\]

3 standard neutrinos + \[\Delta N_{\nu}\] neutrinos with mass \[m_s\].

Contribute to the Rel/NR energy densities.
Results

New massive neutrinos?

3 standard neutrinos  + \( \Delta N_\nu \) neutrinos with mass \( m_s \).

allowed because \( \nu_s \rightsquigarrow \text{CDM} \)

excluded as WDM (Lyman-alpha + LSS)

excluded by bounds on \( N_\nu \)

Standard cosmology

Extra freely-streaming particles

90, 99, 99.9% CL (2 dof)

allowed because \( \nu_s \rightsquigarrow \text{CDM} \)
Results

New massive, sticky particles?
Results

New massive, sticky particles?

\[ \dot{\Theta} + i k \mu \Theta = -\dot{\Phi} - i k \mu \Psi - \tau [\Theta_0 - \Theta + \mu \nu_b - 1/2 P_2(\mu) \Pi] \]
\[ \dot{\Theta}_P + i k \mu \Theta_P = -\tau [\Theta_P + 1/2 (1 - P_2(\mu)) \Pi] \]

\[ \dot{\delta}_{dm} + i k \nu_{dm} = -3 \dot{\Phi} \]
\[ \dot{\nu}_{dm} - \frac{\dot{a}}{a} \nu_{dm} = -i k \Psi \]

\[ \dot{\delta}_b + i k \nu_b = -3 \dot{\Phi} \]
\[ \dot{\nu}_b - \frac{\dot{a}}{a} \nu_b = -i k \Psi + \frac{\tau}{R} [\nu_b + 3i \Theta_1] \]

\[ \dot{N} + i \frac{q_\nu}{E_\nu} k \mu N = -\dot{\Phi} - i \frac{E_\nu}{q_\nu} k \mu \Psi \]

\[ \dot{\delta}_x = -(1 + w)(3 \dot{\Phi} + i k \nu_x) \]
\[ \dot{\nu}_x = -i k \Psi + \frac{\dot{a}}{a} (1 - 3w) i \nu_x - \frac{w}{1+w} i k \delta_x \]

\[ k^2 \dot{\Phi} + 3 \frac{\dot{a}}{a} (\dot{\Phi} - \Psi \frac{\dot{a}}{a}) = 4\pi G_N a^2 [\rho_m \delta_m + 4 \rho_r \delta_r] \]
\[ k^2 (\Phi + \Psi) = -32\pi G_N a^2 \rho_r \Theta_{r,2} \]

CMB Power spectrum

Massive particles, interacting among themselves (i.e. non freely streaming).

\[ \Delta N_{\nu, m_s} \]

A fluid defined by \( \delta_x, \nu_x \),

with \( w = 1/3 \) when rel,

\( w = 0 \) when NR.

Contribute to the Rel/NR energy densities.
Results

New massive, sticky particles?

3 standard neutrinos + $\Delta N_\nu$ with mass $m_s$, interacting among themselves

$\nu_s$ as interacting CDM are more constrained

Bottom Line: Cosmology constrains extra sterile neutrinos (freely-streaming or interacting): they better be few and light.
Can some neutrinos be sticky?
Results

Can some neutrinos be sticky?

\[ \dot{\Theta} + ik\mu \Theta = -\dot{\Phi} - ik\mu \Psi - \dot{\tau} \left[ \Theta_0 - \Theta + \mu \nu_b - 1/2 \mathcal{P}_2(\mu) \Pi \right] \]
\[ \dot{\Theta}_P + ik\mu \Theta_P = -\dot{\tau} \left[ \Theta_P + 1/2 (1 - \mathcal{P}_2(\mu)) \Pi \right] \]

\[ \dot{\delta}_{dm} + ik\nu_{dm} = -3\dot{\Phi} \quad \text{dark matter} \]
\[ \dot{v}_{dm} + \frac{\dot{a}}{a} v_{dm} = -i k \Psi \]

\[ \dot{\delta}_b + ik\nu_b = -3\dot{\Phi} \quad \text{baryons} \]
\[ \dot{v}_b + \frac{\dot{a}}{a} v_b = -i k \Psi + \frac{\dot{\tau}}{R} [v_b + 3i\Theta_1] \]

\[ \dot{N} + i \frac{q_\nu}{E_\nu} k\mu N = -\dot{\Phi} - i \frac{E_\nu}{q_\nu} k\mu \Psi \quad \text{neutrinos} \]
\[ \dot{\delta}_x + i \frac{4}{3} k\nu_x = -4\dot{\Phi} \quad \text{extra} \]
\[ \dot{v}_x + \frac{i}{4} k\delta_x = -i k \Psi \]

\[ k^2 \Phi + 3 \frac{\dot{a}}{a} \left( \dot{\Phi} - \Psi \frac{\dot{a}}{a} \right) = 4\pi G_N a^2 \left[ \rho_m \delta_m + 4\rho_r \delta_r \right] \]
\[ k^2 (\Phi + \Psi) = -32\pi G_N a^2 \rho_r \Theta_{r,2} \]

Neutrinos interacting with extra particles such that free-streaming is prevented.

\[ g \nu\nu \varphi \]
\[ g' \nu\nu_s \varphi \]

interacting particles contribute to Rel energy density.

\[ N_{\nu}^{\text{norm}} \]
\[ N_{\nu}^{\text{int}} + \frac{4}{7} N_\phi \]

freely-streaming neutrinos
Sticky neutrinos don’t stream out of gravitational wells: contribute power to CMB. For massless neutrinos the effect on $P(k)$ is minor.

Quantitatively: (Friedland et al. 2007)

$$\left\{ \frac{\Delta C_\ell}{C_\ell}, \Delta \ell \right\} \approx -\{0.53, 57\} \frac{\rho_{\text{free}}}{\rho_{\text{free}} + \rho_{\text{sticky}} + \rho_\gamma}$$

[Hannestad, JCAP 2005]
[Bell, Pierpaoli, Sigurdson, PRD73 (2006)]
Results

Can some neutrinos be **sticky**?

- **Standard cosmology**
  - $\sim 1$ sticky $\nu$ allowed
    (@ 99% CL, global fit)
  - 3 sticky $\nu$ excluded
    (at 5\sigma)

- **Planck** will greatly improve
  (will test 1 sticky $\nu$ at 4)

[see also Bell, Pierpaoli, Sigurdson, PRD73 (2006)]

[Friedland, Zurek, Bashinsky(2007)]
Results

Can all neutrinos be sticky?

Massive neutrinos and massless boson: $m_\nu$

- e.g. Neutrinoless Universe [Beacom et al. PRL '04]
- e.g. Mass Varying Neutrinos, [Fardon et al. JCAP '04]
- Late Neutrino mass models [Chacko et al. PRD '04]

Massless neutrinos and massive boson: $m_\phi$

- disfavored at 3 to 5 $\sigma$
- [see also Hannestad, JCAP 2004]

Bottom Line: Cosmology strongly disfavors fully interacting (non-freely streaming) neutrinos.
Conclusions & Messages

- Cosmology is a sensitive probe of neutrinos and possible new light particles; let’s put at work the formalism (and a new code) for cosmological perturbation to extract the most from the full cosmo dataset.

- Cosmology gives **dominant bound on** $\sum m_{\nu_i}$; the bound tightens combining relatively less safe datasets.

- Cosmology seems to suggest **5 neutrinos** (2 extra); but Ly-alpha are mainly driving the suggestion.

  The massive extra neutrino of **LSND** was already **strongly disfavored**.

- Cosmology disfavors at various degrees neutrino interactions and other light particles: neutrinos **ought to free-stream**.

- Future observations will be powerful probes.
Extra slides
Lyman-alpha forest

Distant quasar light, redshifted and absorbed at Ly-α frequency by intervening matter, allows to reconstruct matter distribution along the line of sight. But: systematics and uncertainties

Skepticism on Lyman-α: - very complicated measurement and analysis (from flux to matter spectra), different groups disagree (even on same data) - non linearities - HMD simulations don’t include neutrinos
Comparing our code

Our analysis:

WMAP Science Team analysis:

[Spergel et al. WMAP 3yr results '05]

agreement is at few % level and within current precision of data
Neutrinos in the Cosmo

LEPTONS

Neutrino Properties

SUM OF THE NEUTRINO MASSES, $m_{\text{tot}}$

(Defined in the above note), of effectively stable neutrinos (i.e., those with mean lives greater than or equal to the age of the universe). These papers assumed Dirac neutrinos. When necessary, we have generalized the results reported so they apply to $m_{\text{tot}}$. For other limits, see SZALAY 76, VYSOTSKY 77, BERNSTEIN 81, FREESE 84, SCHRAMM 84, and COWSIK 85.

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(Number from Particle Data Book 2008)

Number of Neutrino Types

The neutrinos referred to in this section are those of the Standard $SU(2) \times U(1)$ Electroweak Model possibly extended to allow nonzero neutrino masses. Light neutrinos are those with $m < m_Z/2$. The limits are on the number of neutrino mass eigenstates, including $\nu_1$, $\nu_2$, and $\nu_3$.

Limits from Astrophysics and Cosmology

Number of Light $\nu$ Types

(“light” means < about 1 MeV). See also OLIVE 81. For a review of limits based on Nucleosynthesis, Supernovae, and also on terrestial experiments, see DENEGRI 90. Also see “Big-Bang Nucleosynthesis” in this Review.

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<td>&lt; 2.7</td>
<td>95</td>
<td>WANG 02</td>
<td>COSM CMB</td>
<td></td>
</tr>
<tr>
<td>&lt; 5.5</td>
<td>95</td>
<td>FUKUGITA 00</td>
<td>COSM</td>
<td></td>
</tr>
<tr>
<td>&lt; 7.2</td>
<td>95</td>
<td>CROFT 99</td>
<td>ASTR Ly α power spec</td>
<td></td>
</tr>
<tr>
<td>&lt; 180</td>
<td>95</td>
<td>SZALAY 74</td>
<td>COSM</td>
<td></td>
</tr>
<tr>
<td>&lt; 132</td>
<td>95</td>
<td>COWSIK 72</td>
<td>COSM</td>
<td></td>
</tr>
<tr>
<td>&lt; 280</td>
<td>95</td>
<td>MARX 72</td>
<td>COSM</td>
<td></td>
</tr>
<tr>
<td>&lt; 400</td>
<td>95</td>
<td>GERSHTEIN 66</td>
<td>COSM</td>
<td></td>
</tr>
</tbody>
</table>

(from Particle Data Book 2008)
On neutrino masses

present bounds

future sensitivities

Legenda: the bound or measurement will fall somewhere in the colored box; “where it’ll fall exactly” depends on the author, the experiment considered, priors, the weather...

best summary reference: Lesgourgues, Pastor review
On neutrino masses

FIG. 5: Response of the four reduced CMB observables to the variation of $\omega_\nu$. The isolated points show the values at $\omega_\nu = 0$, which do not connect to the $\omega_\nu \neq 0$ values smoothly.

Ichikawa et al, 2004
Degeneracies

$m_\nu$ effect can be cancelled by $w < -1$. (SNIa data allow less $\Omega_\Lambda$, hence more $\Omega_m$, if $w < -1$; more $\Omega_m$ brings back up the $P(k)$)

or by low $\sigma_8$

Large $N_\nu$ can be cancelled by large $\Omega_m$ or $h$


[back to Nnu]
Degeneracies

\[ \sum m_\nu \text{ will not be forever degenerate with other parameters:} \]

Julien Lesgourgues, talks in 2007

- Massless neutrinos
- Massive neutrinos
- Primordial tilt \( n \)
- Running tilt

Massive neutrinos show a step-like suppression as redshift increases.

DE+CDM+m

Scale-dependent linear growth factor

J.Lesgourgues
Degeneracies

\[ \sum m_\nu \text{ will not be forever degenerate with other parameters:} \]

Julien Lesgourges, talks in 2007

Planck + precision Ly-\(\alpha\):  

Planck alone

Planck + very precise Ly-\(\alpha\)

Planck (with lensing extraction):

\[ f_\nu (\propto \sum m_\nu) \]

\(\sigma_8\)