

LPT Orsay - 21 February 2008

Neutrino properties from Cosmology: the usual and the less usual

Marco Cirelli
(CNRS, IPhT-CEA/Saclay)

with A.Strumia (Pisa)

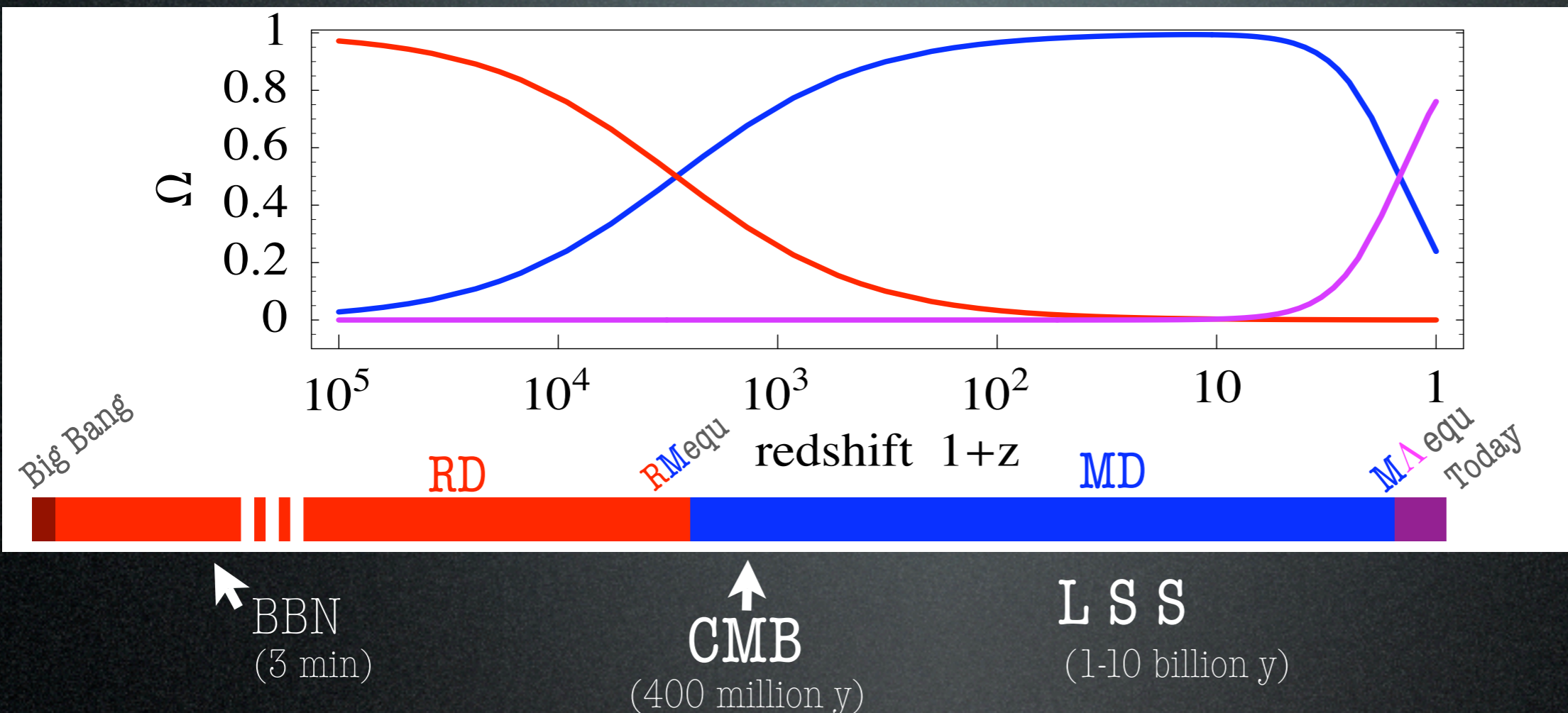
astro-ph/0607086
JCAP 12(2006)013

CONTENTS

- Introduction
- Formalism
- Data set
- Computational tool
- Results

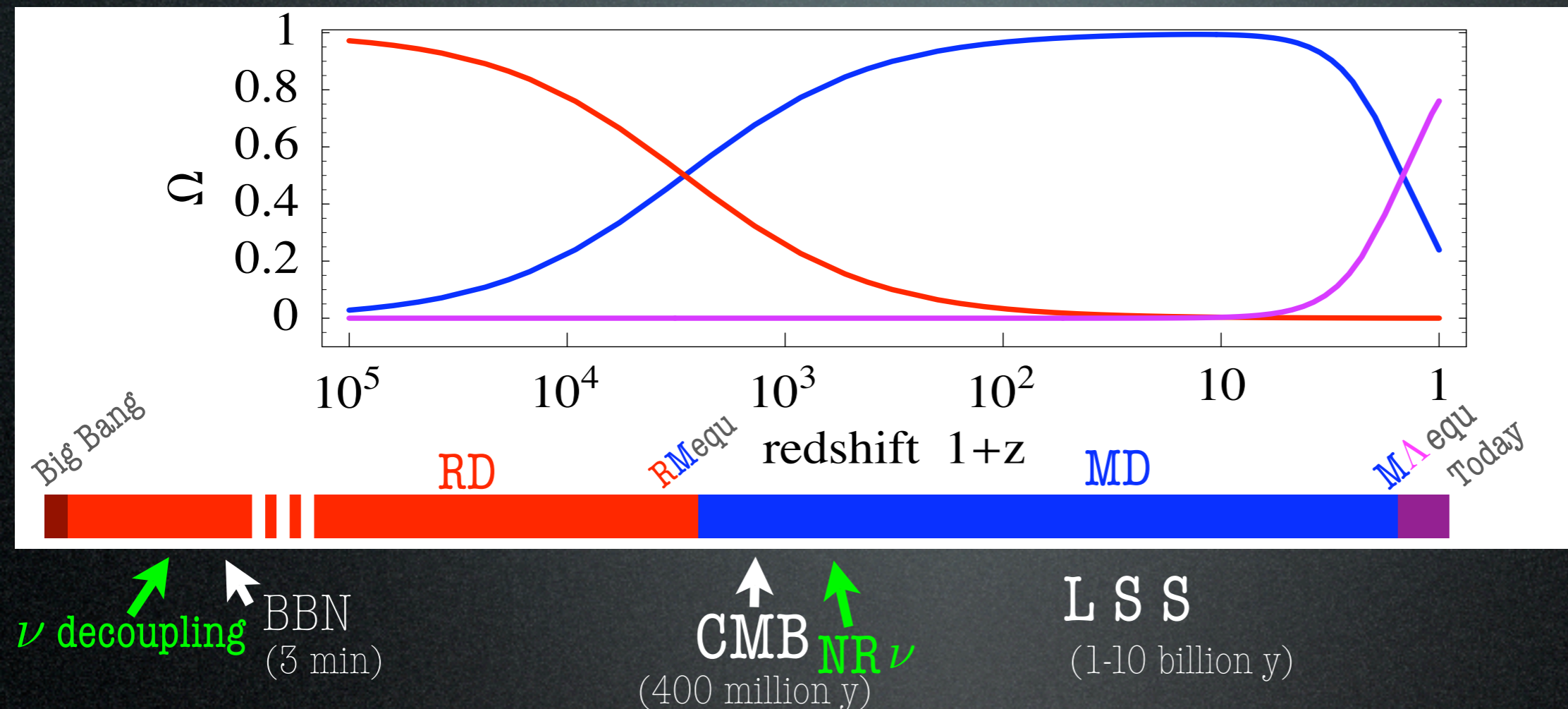
Neutrinos in the Cosmo

The Universe is made of: radiation, matter (DM+b+e), dark energy



Neutrinos in the Cosmo

The Universe is made of: radiation, matter (DM+b+e), dark energy and neutrinos



Neutrinos are significant because:

- main component of the **rel energy density** that sets expansion rate of the Universe
- (ordinary neutrinos have a mass, so) turn from **Rel to NRel** at a crucial time
- may free-stream or **interact** among themselves, or with new light particles

Neutrinos in the Cosmo

So what “neutrinos”?



3 ordinary,
SM neutrinos

extra light degrees of freedom,
very weakly coupled to SM forces

So what properties are probed by cosmology?

- neutrino **number**
- total neutrino **mass**
- non-conventional **interactions**

What are the relevant cosmological probes?

- **BBN** ($T \sim \text{MeV}$, flavor is important, primordial plasma)
- later cosmology i.e. **CMB+LSS** ($T \lesssim \text{eV}$, $\approx m_\nu$, gravity is the only force)

Cosmological data are (mostly) *not* sensitive to:

θ_{active} , $m_{1,2,3}$ (or $\Delta m_{\text{active}}^2$), CP —violation...

Neutrinos in the Cosmo

So what “neutrinos”?



3 ordinary,
SM neutrinos

extra light degrees of freedom,
very weakly coupled to SM forces

So what properties are probed by cosmology?

- neutrino **number**
- total neutrino **mass**
- non-conventional **interactions**

What are the relevant cosmological probes?

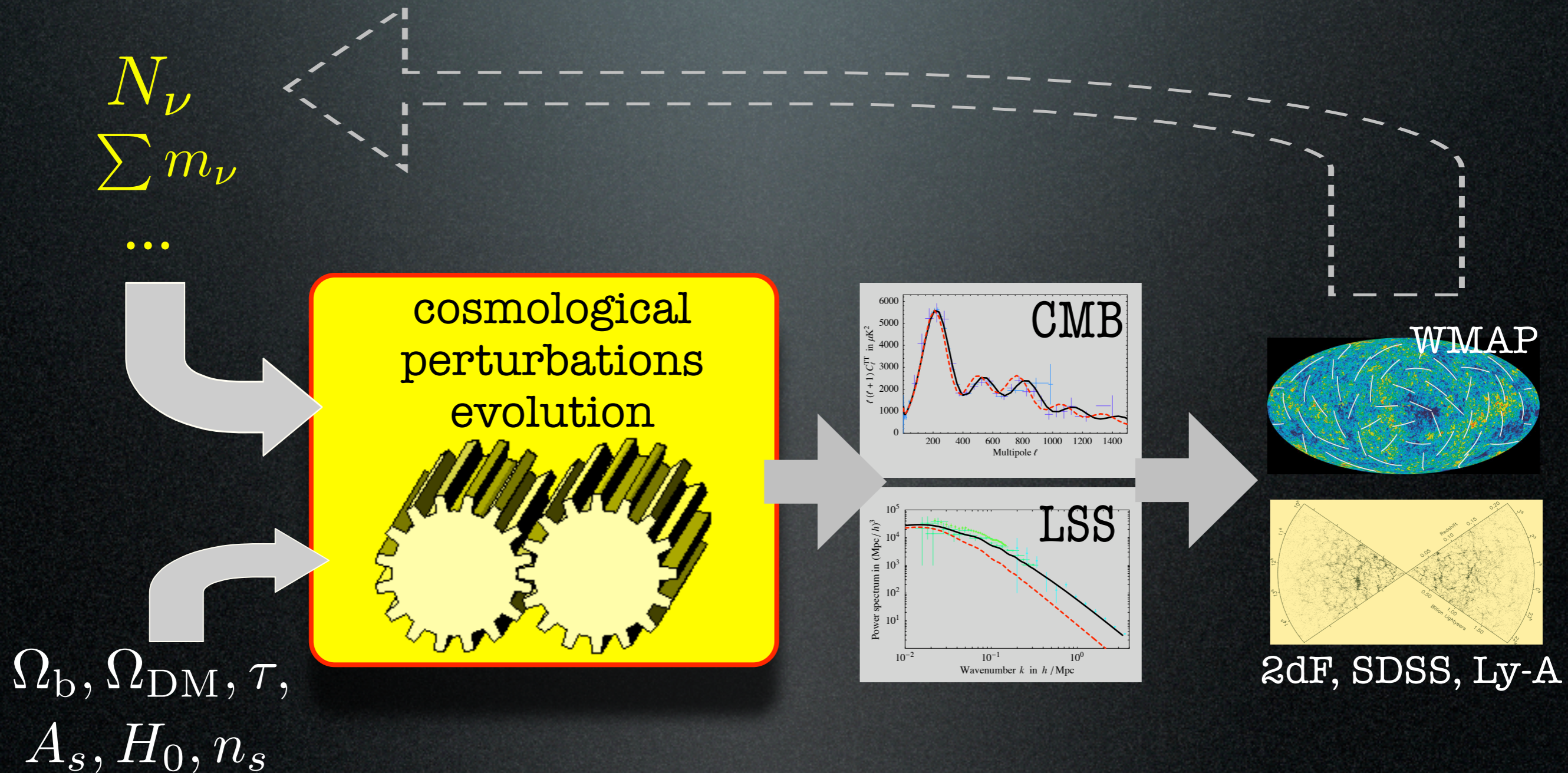
- **BBN** ($T \sim \text{MeV}$, flavor is important, primordial plasma)
- later cosmology i.e. **CMB+LSS** ($T \lesssim \text{eV}$, $\approx m_\nu$, gravity is the only force)

Cosmological data are (mostly) *not* sensitive to:

θ_{active} , $m_{1,2,3}$ (or $\Delta m_{\text{active}}^2$), CP —violation...

Neutrinos in CMB+LSS

Neutrinos affect (indirectly, i.e. gravitationally) the evolution of cosmological perturbations in radiation and matter.



Formalism

Dodelson's (Chicago, 2003)
notations

(=cosmological perturbation theory in one slide)

$$\left. \begin{aligned} \dot{\Theta} + ik\mu\Theta &= -\dot{\Phi} - ik\mu\Psi - \dot{\tau} \left[\Theta_0 - \Theta + \mu v_b - 1/2 \mathcal{P}_2(\mu)\Pi \right] \\ \dot{\Theta}_P + ik\mu\Theta_P &= -\dot{\tau} \left[\Theta_P + 1/2(1 - \mathcal{P}_2(\mu))\Pi \right] \end{aligned} \right\} \text{photons}$$

$\dot{\tau} = d\tau/d\eta = -n_e\sigma_T a \quad \Pi = \Theta_2 + \Theta_{P2} + \Theta_{P0}$

$$\left. \begin{aligned} \dot{\delta}_{\text{dm}} + ikv_{\text{dm}} &= -3\dot{\Phi} \\ \dot{v}_{\text{dm}} + \frac{\dot{a}}{a}v_{\text{dm}} &= -ik\Psi \end{aligned} \right\} \text{dark matter}$$

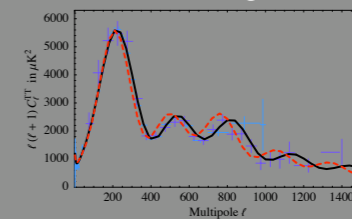
$$\left. \begin{aligned} \dot{\delta}_b + ikv_b &= -3\dot{\Phi} \\ \dot{v}_b + \frac{\dot{a}}{a}v_b &= -ik\Psi + \frac{\dot{\tau}}{R} \left[v_b + 3i\Theta_1 \right] \end{aligned} \right\} \text{baryons}$$

$R = 3\rho_b^0/4\rho_\gamma^0$

$$\left. \begin{aligned} \dot{\mathcal{N}} + i\frac{q_\nu}{E_\nu}k\mu\mathcal{N} &= -\dot{\Phi} - i\frac{E_\nu}{q_\nu}k\mu\Psi \end{aligned} \right\} \text{neutrinos}$$

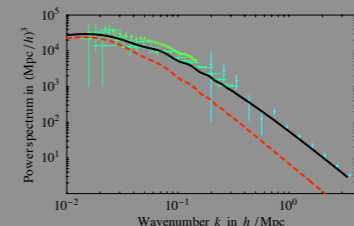
CMB Power spectrum

$$C_\ell \propto \int dk [\dots] \Theta_\ell(k)$$



Matter Power spect.

$$P(k) \propto \langle \delta_m(k)^2 \rangle$$



$$\left. \begin{aligned} k^2\Phi + 3\frac{\dot{a}}{a}\left(\dot{\Phi} - \Psi\frac{\dot{a}}{a}\right) &= 4\pi G_N a^2 [\rho_m\delta_m + 4\rho_r\delta_r] \\ k^2(\Phi + \Psi) &= -32\pi G_N a^2 \rho_r \Theta_{r,2} \end{aligned} \right\} \text{metric}$$

Formalism

(=cosmological perturbation theory in one slide)

$$\left. \begin{aligned} \dot{\Theta} + ik\mu\Theta &= -\dot{\Phi} - ik\mu\Psi - \dot{\tau} \left[\Theta_0 - \Theta + \mu v_b - 1/2 \mathcal{P}_2(\mu)\Pi \right] \\ \dot{\tau} &= d\tau/d\eta = -n_e\sigma_T a & \Pi &= \Theta_2 + \Theta_{P2} + \Theta_{P0} \end{aligned} \right\} \text{photons}$$

$$\Theta = \frac{\delta T}{T} \quad f(\vec{x}, \vec{p}) = \frac{1}{e^{\frac{p}{T+\delta T}} - 1}$$

Fourier: $\Theta(\vec{x}, \vec{p}, t) \longrightarrow \Theta(k, \mu, \eta) \quad \mu = \hat{k} \cdot \hat{p}$

Expand in multipoles:

$$\Theta_\ell(k, \eta) = \frac{1}{(-1)^\ell} \int_{-1}^1 d\mu \frac{1}{2} \mathcal{P}(\mu) \Theta(k, \mu, \eta)$$

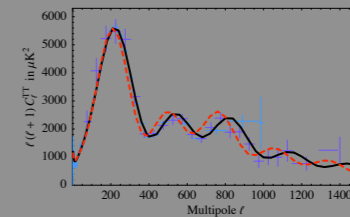
$$\left. \dot{v}_b + \frac{\dot{a}}{a} v_b = -ik\Psi + \frac{\dot{\tau}}{R} [v_b + 3i\Theta_1] \right\} \text{baryons}$$

$$\left. \dot{\mathcal{N}} + i \frac{q_\nu}{E_\nu} k\mu \mathcal{N} = -\dot{\Phi} - i \frac{E_\nu}{q_\nu} k\mu \Psi \right\} \text{neutrinos}$$

$$\left. \begin{aligned} k^2 \Phi + 3 \frac{\dot{a}}{a} \left(\dot{\Phi} - \Psi \frac{\dot{a}}{a} \right) &= 4\pi G_N a^2 [\rho_m \delta_m + 4\rho_r \delta_r] \\ k^2 (\Phi + \Psi) &= -32\pi G_N a^2 \rho_r \Theta_{r,2} \end{aligned} \right\} \text{metric}$$

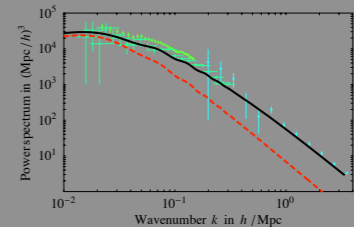
CMB Power spectrum

$$C_\ell \propto \int dk [\dots] \Theta_\ell(k)$$



Matter Power spect.

$$P(k) \propto \langle \delta_m(k)^2 \rangle$$



Formalism

(=cosmological perturbation theory in one slide)

$$\left. \begin{aligned} \dot{\Theta} + ik\mu\Theta &= -\dot{\Phi} - ik\mu\Psi - \dot{\tau} \left[\Theta_0 - \Theta + \mu v_b - 1/2 \mathcal{P}_2(\mu)\Pi \right] \\ \dot{\Theta}_P + ik\mu\Theta_P &= -\dot{\tau} \left[\Theta_P + 1/2(1 - \mathcal{P}_2(\mu))\Pi \right] \end{aligned} \right\} \text{photons}$$

$\dot{\tau} = d\tau/d\eta = -n_e\sigma_T a \quad \Pi = \Theta_2 + \Theta_{P2} + \Theta_{P0}$

$$\left. \begin{aligned} \dot{\delta}_{\text{dm}} + ikv_{\text{dm}} &= -3\dot{\Phi} \\ \dot{v}_{\text{dm}} + \frac{\dot{a}}{a}v_{\text{dm}} &= -ik\Psi \end{aligned} \right\} \text{dark matter}$$

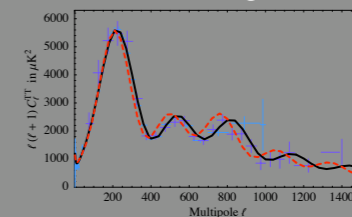
$$\delta_{\text{dm}} = \frac{\delta\rho_{\text{dm}}}{\rho_{\text{dm}}} \quad \rho_{\text{dm}}(\vec{x}, t) = \rho_{\text{dm}}^0 \left(1 + \delta_{\text{dm}}(\vec{x}, t) \right)$$

and velocity v_{dm}

Fourier: $\delta_{\text{dm}}(\vec{x}, t) \longrightarrow \delta_{\text{dm}}(k, \eta)$
 $v_{\text{dm}}(\vec{x}, t) \longrightarrow v_{\text{dm}}(k, \eta)$

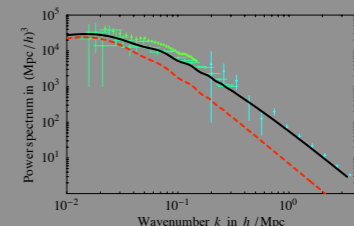
CMB Power spectrum

$$C_\ell \propto \int dk [\dots] \Theta_\ell(k)$$



Matter Power spect.

$$P(k) \propto \langle \delta_m(k)^2 \rangle$$



$$\left. \begin{aligned} k^2\Phi + 3\frac{\dot{a}}{a}\left(\dot{\Phi} - \Psi\frac{\dot{a}}{a}\right) &= 4\pi G_N a^2 [\rho_m\delta_m + 4\rho_r\delta_r] \\ k^2(\Phi + \Psi) &= -32\pi G_N a^2 \rho_r \Theta_{r,2} \end{aligned} \right\} \text{metric}$$

Formalism

(=cosmological perturbation theory in one slide)

$$\left. \begin{aligned} \dot{\Theta} + ik\mu\Theta &= -\dot{\Phi} - ik\mu\Psi - \dot{\tau} \left[\Theta_0 - \Theta + \mu v_b - 1/2 \mathcal{P}_2(\mu)\Pi \right] \\ \dot{\Theta}_P + ik\mu\Theta_P &= -\dot{\tau} \left[\Theta_P + 1/2(1 - \mathcal{P}_2(\mu))\Pi \right] \end{aligned} \right\} \text{photons}$$

$\dot{\tau} = d\tau/d\eta = -n_e\sigma_T a \quad \Pi = \Theta_2 + \Theta_{P2} + \Theta_{P0}$

$$\left. \begin{aligned} \dot{\delta}_{\text{dm}} + ikv_{\text{dm}} &= -3\dot{\Phi} \\ \dot{v}_{\text{dm}} + \frac{\dot{a}}{a}v_{\text{dm}} &= -ik\Psi \end{aligned} \right\} \text{dark matter}$$

$$\left. \begin{aligned} \dot{\delta}_b + ikv_b &= -3\dot{\Phi} \\ \dot{v}_b + \frac{\dot{a}}{a}v_b &= -ik\Psi + \frac{\dot{\tau}}{R} \left[v_b + 3i\Theta_1 \right] \end{aligned} \right\} \text{baryons}$$

$R = 3\rho_b^0/4\rho_\gamma^0$

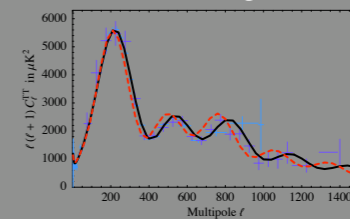
$\delta_b(k, \eta)$
 $v_b(k, \eta)$

Thomson scattering

 $e^- \gamma \longleftrightarrow e^- \gamma$

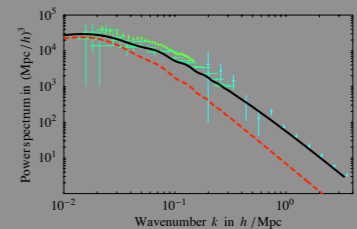
CMB Power spectrum

$$C_\ell \propto \int dk [\dots] \Theta_\ell(k)$$



Matter Power spect.

$$P(k) \propto \langle \delta_m(k)^2 \rangle$$



$$\left. \begin{aligned} k^2 \Phi + 3 \frac{\dot{a}}{a} \left(\dot{\Phi} - \Psi \frac{\dot{a}}{a} \right) &= 4\pi G_N a^2 [\rho_m \delta_m + 4\rho_r \delta_r] \\ k^2 (\Phi + \Psi) &= -32\pi G_N a^2 \rho_r \Theta_{r,2} \end{aligned} \right\} \text{metric}$$

Formalism

(=cosmological perturbation theory in one slide)

$$\left. \begin{aligned} \dot{\Theta} + ik\mu\Theta &= -\dot{\Phi} - ik\mu\Psi - \dot{\tau} \left[\Theta_0 - \Theta + \mu v_b - 1/2 \mathcal{P}_2(\mu)\Pi \right] \\ \dot{\Theta}_P + ik\mu\Theta_P &= -\dot{\tau} \left[\Theta_P + 1/2(1 - \mathcal{P}_2(\mu))\Pi \right] \end{aligned} \right\} \text{photons}$$

$\dot{\tau} = d\tau/d\eta = -n_e\sigma_T a \quad \Pi = \Theta_2 + \Theta_{P2} + \Theta_{P0}$

$$\left. \begin{aligned} \dot{\delta}_{\text{dm}} + ikv_{\text{dm}} &= -3\dot{\Phi} \\ \dot{v}_{\text{dm}} + \frac{\dot{a}}{a}v_{\text{dm}} &= -ik\Psi \end{aligned} \right\} \text{dark matter}$$

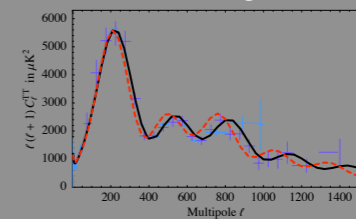
scalar metric perturbations: $g_{\mu\nu} = \eta_{\mu\nu} + \delta\eta_{\mu\nu}(\Psi, \Phi)$

$$g_{\mu\nu} = \begin{pmatrix} -1 - 2\Psi & 0 & 0 & 0 \\ 0 & a^2(1 + 2\Phi) & 0 & 0 \\ 0 & 0 & a^2(1 + 2\Phi) & 0 \\ 0 & 0 & 0 & a^2(1 + 2\Phi) \end{pmatrix}$$

Fourier: $\Psi(\vec{x}, t) \longrightarrow \Psi(k, \eta)$
 $\Phi(\vec{x}, t) \longrightarrow \Phi(k, \eta)$

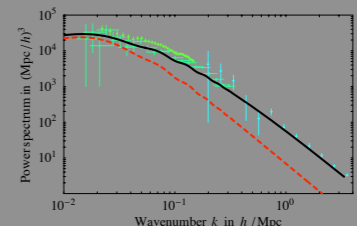
CMB Power spectrum

$$C_\ell \propto \int dk [\dots] \Theta_\ell(k)$$



Matter Power spect.

$$P(k) \propto \langle \delta_m(k)^2 \rangle$$



$$\left. \begin{aligned} k^2\Phi + 3\frac{\dot{a}}{a}\left(\dot{\Phi} - \Psi\frac{\dot{a}}{a}\right) &= 4\pi G_N a^2 [\rho_m \delta_m + 4\rho_r \delta_r] \\ k^2(\Phi + \Psi) &= -32\pi G_N a^2 \rho_r \Theta_{r,2} \end{aligned} \right\} \text{metric}$$

Formalism

(=cosmological perturbation theory in one slide)

$$\left. \begin{aligned} \dot{\Theta} + ik\mu\Theta &= -\dot{\Phi} - ik\mu\Psi - \dot{\tau} \left[\Theta_0 - \Theta + \mu v_b - 1/2 \mathcal{P}_2(\mu)\Pi \right] \\ \dot{\Theta}_P + ik\mu\Theta_P &= -\dot{\tau} \left[\Theta_P + 1/2(1 - \mathcal{P}_2(\mu))\Pi \right] \end{aligned} \right\} \text{photons}$$

$\dot{\tau} = d\tau/d\eta = -n_e\sigma_T a \quad \Pi = \Theta_2 + \Theta_{P2} + \Theta_{P0}$

$$\left. \begin{aligned} \dot{\delta}_{\text{dm}} + ikv_{\text{dm}} &= -3\dot{\Phi} \\ \dot{v}_{\text{dm}} + \frac{\dot{a}}{a}v_{\text{dm}} &= -ik\Psi \end{aligned} \right\} \text{dark matter}$$

$$\left. \begin{aligned} \dot{\delta}_b + ikv_b &= -3\dot{\Phi} \\ \dot{v}_b + \frac{\dot{a}}{a}v_b &= -ik\Psi + \frac{\dot{\tau}}{R} \left[v_b + 3i\Theta_1 \right] \end{aligned} \right\} \text{baryons}$$

$R = 3\rho_b^0/4\rho_\gamma^0$

$$\left. \begin{aligned} \dot{\mathcal{N}} + i\frac{q_\nu}{E_\nu}k\mu\mathcal{N} &= -\dot{\Phi} - i\frac{E_\nu}{q_\nu}k\mu\Psi \end{aligned} \right\} \text{neutrinos}$$

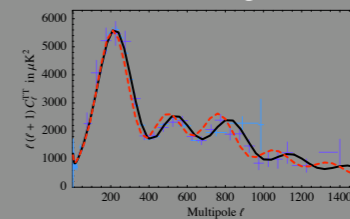
massless or massive neutrinos $E_\nu = \sqrt{p_\nu^2 + m_\nu^2}$

Fourier: $\mathcal{N}(\vec{x}, \vec{p}, t) \longrightarrow \mathcal{N}(k, \mu, \eta)$

Expand in multipoles: $\mathcal{N}_\ell(k, \mu, \eta)$

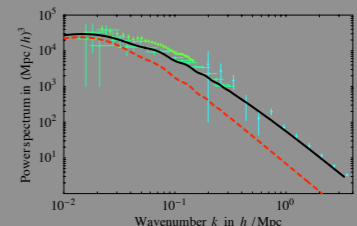
CMB Power spectrum

$$C_\ell \propto \int dk [\dots] \Theta_\ell(k)$$



Matter Power spect.

$$P(k) \propto \langle \delta_m(k)^2 \rangle$$



$$k^2 (\Phi + \Psi) = -32\pi G_N a^2 \rho_r \Theta_{r,2}$$

Formalism

(=cosmological perturbation theory in one slide)

$$\left. \begin{aligned} \dot{\Theta} + ik\mu\Theta &= -\dot{\Phi} - ik\mu\Psi - \dot{\tau} \left[\Theta_0 - \Theta + \mu v_b - 1/2 \mathcal{P}_2(\mu)\Pi \right] \\ \dot{\Theta}_P + ik\mu\Theta_P &= -\dot{\tau} \left[\Theta_P + 1/2(1 - \mathcal{P}_2(\mu))\Pi \right] \end{aligned} \right\} \text{photons}$$

$\dot{\tau} = d\tau/d\eta = -n_e\sigma_T a \quad \Pi = \Theta_2 + \Theta_{P2} + \Theta_{P0}$

$$\left. \begin{aligned} \dot{\delta}_{\text{dm}} + ikv_{\text{dm}} &= -3\dot{\Phi} \\ \dot{v}_{\text{dm}} + \frac{\dot{a}}{a}v_{\text{dm}} &= -ik\Psi \end{aligned} \right\} \text{dark matter}$$

$$\left. \begin{aligned} \dot{\delta}_b + ikv_b &= -3\dot{\Phi} \\ \dot{v}_b + \frac{\dot{a}}{a}v_b &= -ik\Psi + \frac{\dot{\tau}}{R} \left[v_b + 3i\Theta_1 \right] \end{aligned} \right\} \text{baryons}$$

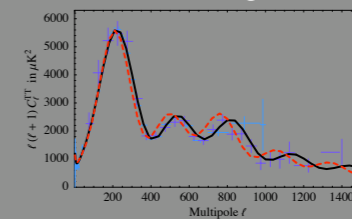
$R = 3\rho_b^0/4\rho_\gamma^0$

$$\left. \begin{aligned} \dot{\mathcal{N}} + i\frac{q_\nu}{E_\nu}k\mu\mathcal{N} &= -\dot{\Phi} - i\frac{E_\nu}{q_\nu}k\mu\Psi \end{aligned} \right\} \text{neutrinos}$$

$$\left. \begin{aligned} k^2\Phi + 3\frac{\dot{a}}{a}\left(\dot{\Phi} - \Psi\frac{\dot{a}}{a}\right) &= 4\pi G_N a^2 [\rho_m\delta_m + 4\rho_r\delta_r] \\ k^2(\Phi + \Psi) &= -32\pi G_N a^2 \rho_r \Theta_{r,2} \end{aligned} \right\} \text{metric}$$

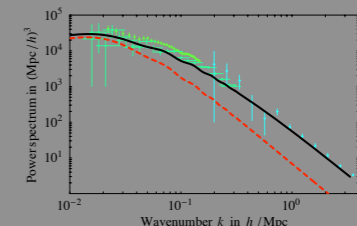
CMB Power spectrum

$$C_\ell \propto \int dk [\dots] \Theta_\ell(k)$$



Matter Power spect.

$$P(k) \propto \langle \delta_m(k)^2 \rangle$$



An application:

e.g. Lesgourgues, Pastor review;
Bond, Efstathiou, Silk, 1980

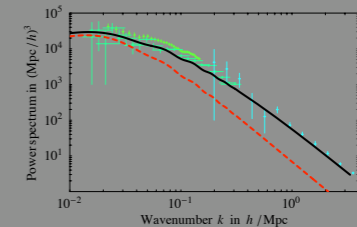
the effect of neutrino mass on the Matter Power Spectrum

- let's follow δ_{dm} during MD (matter perturbations don't grow during RD)

$$\left. \begin{aligned} \dot{\delta}_{\text{dm}} + ikv_{\text{dm}} &= -3\dot{\Phi} \\ \dot{v}_{\text{dm}} + \frac{\dot{a}}{a}v_{\text{dm}} &= -ik\Psi \end{aligned} \right\} \text{dark matter}$$

Matter Power spect.

$$P(k) \propto \langle \delta_{\text{m}}(k)^2 \rangle$$



$$\left. \begin{aligned} k^2\Phi + 3\frac{\dot{a}}{a}\left(\dot{\Phi} - \Psi\frac{\dot{a}}{a}\right) &= 4\pi G_N a^2 [\rho_{\text{m}}\delta_{\text{m}} + 4\rho_{\text{r}}\delta_{\text{r}}] \\ k^2(\Phi + \Psi) &= -32\pi G_N a^2 \rho_{\text{r}}\Theta_{\text{r},2} \end{aligned} \right\} \text{metric}$$

An application:

the effect of neutrino mass on the Matter Power Spectrum

- let's follow δ_{dm} during MD (matter perturbations don't grow during RD)

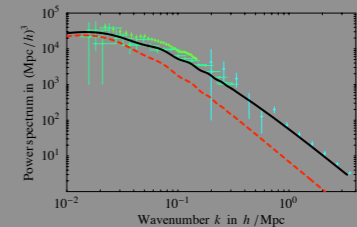
$$\ddot{\delta}_{\text{dm}} + \frac{\dot{a}}{a} \dot{\delta}_{\text{dm}} \simeq k^2 \Phi$$

$$\left. \begin{aligned} \dot{\delta}_{\text{dm}} + ikv_{\text{dm}} &= -3\dot{\Phi} \\ \dot{v}_{\text{dm}} + \frac{\dot{a}}{a} v_{\text{dm}} &= -ik\Psi \end{aligned} \right\} \text{dark matter}$$



Matter Power spect.

$$P(k) \propto \langle \delta_{\text{m}}(k)^2 \rangle$$



$$k^2 \Phi = 4\pi G_N a^2 [\rho_{\text{m}} \delta_{\text{m}} + 4\rho_{\text{r}} \Theta_{\text{r},0} + \frac{3Ha}{K} (i\rho_{\text{m}} v_{\text{m}} + 4\rho_{\text{r}} \Theta_{\text{r},1})]$$

$$\left. \begin{aligned} k^2 \Phi + 3 \frac{\dot{a}}{a} \left(\dot{\Phi} - \Psi \frac{\dot{a}}{a} \right) &= 4\pi G_N a^2 [\rho_{\text{m}} \delta_{\text{m}} + 4\rho_{\text{r}} \delta_{\text{r}}] \\ k^2 (\Phi + \Psi) &= -32\pi G_N a^2 \rho_{\text{r}} \Theta_{\text{r},2} \end{aligned} \right\} \text{metric}$$



An application:

the effect of neutrino mass on the Matter Power Spectrum

- let's follow δ_{dm} during MD (matter perturbations don't grow during RD)

$$\ddot{\delta}_{\text{dm}} + \frac{\dot{a}}{a} \dot{\delta}_{\text{dm}} \simeq 4\pi G_N a^2 \rho_{\text{m}} \delta_{\text{m}} \quad (\text{Newton eq. for } \delta_{\text{dm}}) \quad \left(\cdot = \frac{d}{d\eta} \right)$$

An application:

the effect of neutrino mass on the Matter Power Spectrum

- let's follow δ_{dm} during MD (matter perturbations don't grow during RD)

$$\ddot{\delta}_{\text{dm}} + \frac{\dot{a}}{a} \dot{\delta}_{\text{dm}} \simeq 4\pi G_N a^2 \rho_{\text{m}} \delta_{\text{m}} \quad (\text{Newton eq. for } \delta_{\text{dm}}) \quad \left(\cdot = \frac{d}{d\eta} \right)$$

- with **massless neutrinos**:

$$\text{FRW eq. } H^2 = \frac{8}{3} \pi G_N \rho_{\text{m}} \quad \text{with } \rho_{\text{m}} = (\rho_{\text{dm}} + \rho_{\text{b}}) \propto a^{-3} \quad \Rightarrow \quad a \propto t^{2/3}$$

$$\text{so } \delta_{\text{dm}}'' + \frac{4}{3} \frac{1}{t} \delta_{\text{dm}}' - \frac{2}{3} \frac{1}{t^2} \delta_{\text{dm}} = 0 \quad \Rightarrow \quad \text{growing solution } \delta_{\text{dm}} \propto t^{\frac{2}{3}} \propto a$$

An application:

the effect of neutrino mass on the Matter Power Spectrum

- let's follow δ_{dm} during MD (matter perturbations don't grow during RD)

$$\ddot{\delta}_{\text{dm}} + \frac{\dot{a}}{a} \dot{\delta}_{\text{dm}} \simeq 4\pi G_N a^2 \rho_{\text{m}} \delta_{\text{m}} \quad (\text{Newton eq. for } \delta_{\text{dm}}) \quad \left(\dot{} = \frac{d}{d\eta} \right)$$

- with **massless neutrinos**:

$$\text{FRW eq. } H^2 = \frac{8}{3} \pi G_N \rho_{\text{m}} \quad \text{with } \rho_{\text{m}} = (\rho_{\text{dm}} + \rho_{\text{b}}) \propto a^{-3} \Rightarrow a \propto t^{2/3}$$

$$\text{so } \delta_{\text{dm}}'' + \frac{4}{3} \frac{1}{t} \delta_{\text{dm}}' - \frac{2}{3} \frac{1}{t^2} \delta_{\text{dm}} = 0 \Rightarrow \text{growing solution } \delta_{\text{dm}} \propto t^{\frac{2}{3}} \propto a$$

- with **massive neutrinos**:

$$\text{FRW eq. with } \rho_{\text{m}} = (\rho_{\text{dm}} + \rho_{\text{b}} + \rho_{\nu}) = \rho_{\text{m}}(1 + f_{\nu}), \quad f_{\nu} = \frac{\Omega_{\nu}}{\Omega_{\text{m}}} \Rightarrow a \propto (1 + f_{\nu})^{\frac{1}{3}} t^{2/3}$$

$$\Omega_{\nu} = \sum m_{\nu_i} / 93 \text{ eV}$$

but $\delta_{\nu} = 0$ because neutrinos don't cluster (at $k \gg k_{\text{NR}}$, see below)

i.e. **massive neutrinos contribute to the energy density** of the Universe during MD

but they don't source in the Newton equation for δ_{dm} !

$$\text{thus } \delta_{\text{dm}}'' + \frac{4}{3} \frac{1}{t} \delta_{\text{dm}}' - \frac{2}{3} (1 - f_{\nu}) \frac{1}{t^2} \delta_{\text{dm}} = 0 \Rightarrow \delta_{\text{dm}} \propto t^{\frac{-1 + \sqrt{25 - 24f_{\nu}}}{6}} \propto a^{1 - \frac{3}{5} f_{\nu}}$$

An application:

the effect of neutrino mass on the Matter Power Spectrum

- let's follow δ_{dm} during MD (matter perturbations don't grow during RD)

$$\ddot{\delta}_{\text{dm}} + \frac{\dot{a}}{a} \dot{\delta}_{\text{dm}} \simeq 4\pi G_N a^2 \rho_{\text{m}} \delta_{\text{m}} \quad (\text{Newton eq. for } \delta_{\text{dm}}) \quad \left(\dot{} = \frac{d}{d\eta} \right)$$

- with **massless neutrinos**:

$$\text{FRW eq. } H^2 = \frac{8}{3} \pi G_N \rho_{\text{m}} \quad \text{with } \rho_{\text{m}} = (\rho_{\text{dm}} + \rho_{\text{b}}) \propto a^{-3} \Rightarrow a \propto t^{2/3}$$

$$\text{so } \delta_{\text{dm}}'' + \frac{4}{3} \frac{1}{t} \delta_{\text{dm}}' - \frac{2}{3} \frac{1}{t^2} \delta_{\text{dm}} = 0 \Rightarrow \text{growing solution } \delta_{\text{dm}} \propto t^{\frac{2}{3}} \propto a$$

- with **massive neutrinos**:

$$\text{FRW eq. with } \rho_{\text{m}} = (\rho_{\text{dm}} + \rho_{\text{b}} + \rho_{\nu}) = \rho_{\text{m}}(1 + f_{\nu}), \quad f_{\nu} = \frac{\Omega_{\nu}}{\Omega_{\text{m}}} \Rightarrow a \propto (1 + f_{\nu})^{\frac{1}{3}} t^{2/3}$$

$$\Omega_{\nu} = \sum m_{\nu_i} / 93 \text{ eV}$$

but $\delta_{\nu} = 0$ because neutrinos don't cluster (at $k \gg k_{\text{NR}}$, see below)

i.e. **massive neutrinos contribute to the energy density** of the Universe during MD

but they don't source in the Newton equation for δ_{dm} !

$$\text{thus } \delta_{\text{dm}}'' + \frac{4}{3} \frac{1}{t} \delta_{\text{dm}}' - \frac{2}{3} (1 - f_{\nu}) \frac{1}{t^2} \delta_{\text{dm}} = 0 \Rightarrow \delta_{\text{dm}} \propto t^{\frac{-1 + \sqrt{25 - 24f_{\nu}}}{6}} \propto a^{1 - \frac{3}{5} f_{\nu}}$$

- so effective suppression of the growth is: $\frac{\delta_{\text{dm}}^{(m_{\nu} \neq 0)}}{\delta_{\text{dm}}^{(m_{\nu} = 0)}} \simeq (a_{\text{NR}})^{\frac{3}{5} f_{\nu}}$ (a_{NR} because at that point massless and massive coincide)

or in terms of $P(k) \propto \langle \delta_{\text{dm}} \rangle^2$: $\frac{P^{(m_{\nu} \neq 0)}(k)}{P^{(m_{\nu} = 0)}(k)} \simeq (a_{\text{NR}})^{\frac{6}{5} f_{\nu}}$ modeled by $\boxed{\frac{\Delta P}{P} \simeq -8 f_{\nu}}$

An application:

the effect of neutrino mass on the Matter Power Spectrum

- at what scales is the effect relevant?

The free streaming scale is the distance traveled by a neutrino in a Hubble time

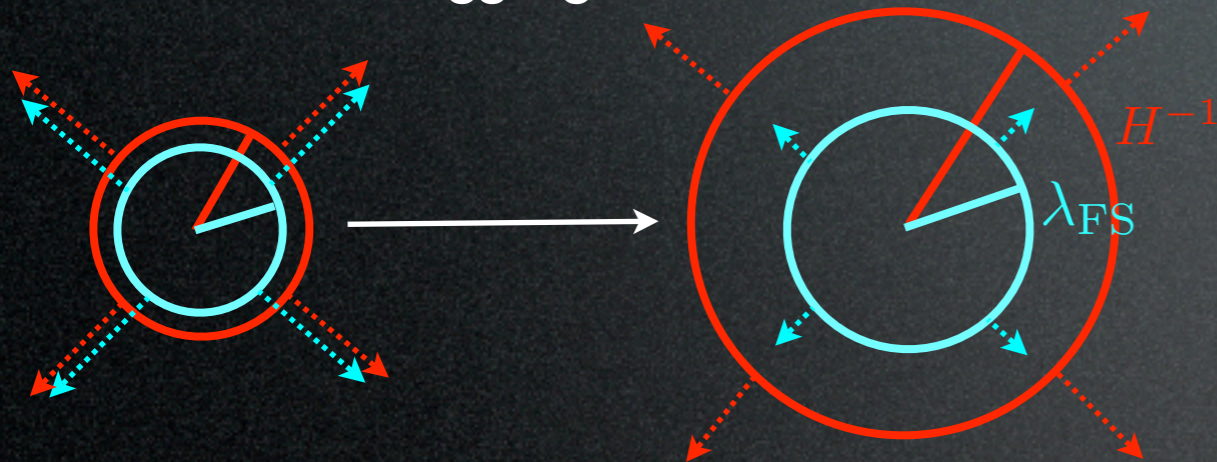
$$\lambda_{\text{FS}}(t) \simeq \frac{v(t)}{H(t)} \quad \leftarrow \text{thermal velocity in units of } c$$

For massive neutrinos $v(t) \simeq \frac{3T_\nu}{m_\nu} \propto \frac{1}{a} \left(\frac{1 \text{ eV}}{m_\nu} \right) \frac{\text{km}}{\text{sec}}$

so that $\lambda_{\text{FS}} \propto \frac{a^{-1}}{\sqrt{\Omega_m a^{-3}}} \left(\frac{1 \text{ eV}}{m_\nu} \right) \frac{\text{km}}{\text{sec}} \propto a^{1/2}$

but the size of the Universe scales as $\frac{1}{H(t)} \propto a^{3/2}$

so λ_{FS} starts lagging behind H^{-1} when ν become NR, at $a_{\text{NR}}^{-1} \simeq 2.1 \cdot 10^3 \frac{m_\nu}{\text{eV}}$
(ie m_ν becomes relevant)



The effect of free streaming affects comoving scales inside $\frac{\lambda_{\text{NR}}}{a} = \frac{\lambda_{\text{FS}}}{a_{\text{NR}}}$

or in terms of $k_{\text{NR}} = 2\pi \left(\frac{\lambda_{\text{NR}}}{a} \right)^{-1}$ at

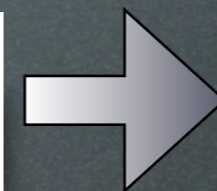
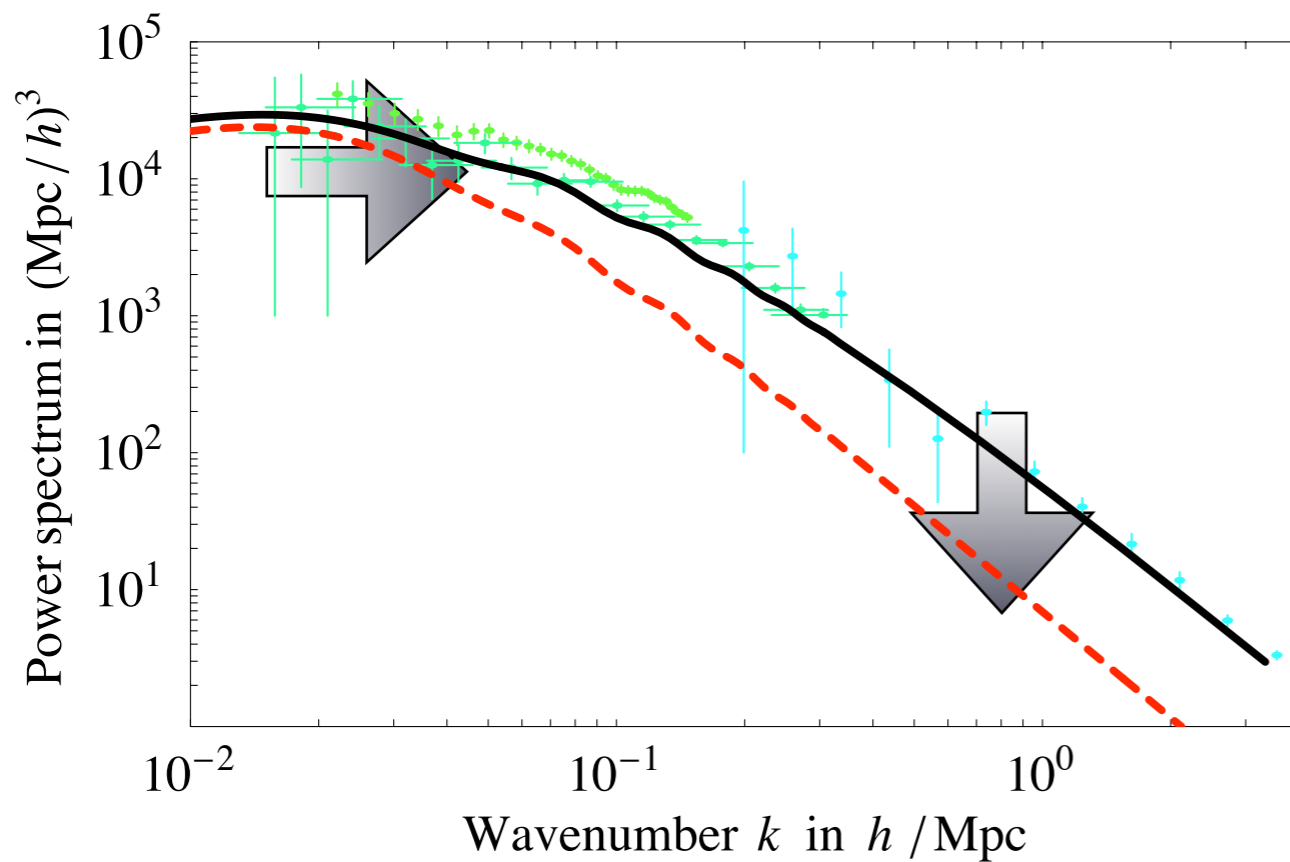
$$k \gg k_{\text{NR}} = 0.018 \Omega_m^{-1/2} \left(\frac{m_\nu}{\text{eV}} \right)^{1/2} h_0 \text{ Mpc}^{-1}$$

An application: the effect of neutrino mass on the Matter Power Spectrum

In summary

— $m_\nu = 0$
- - - $m_\nu = 2 \text{ eV}$

Caveat: plots for illustrative purposes only,
all parameters except m_ν are held fixed.



$$k_{\text{NR}} = 0.018 \Omega_{\text{m}}^{-1/2} \left(\frac{m_\nu}{\text{eV}} \right)^{1/2} h_0 \text{ Mpc}^{-1}$$




$$\frac{\Delta P}{P} \simeq -8 f_\nu \quad \left(f_\nu = \frac{\sum m_{\nu_i} / 93 \text{ eV}}{\Omega_{\text{m}}} \right)$$

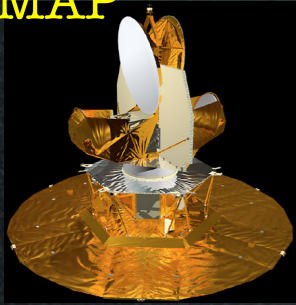
We have the formalism to compute
the effect on cosmological observables.

Let's compare quantitatively
with cosmological data.

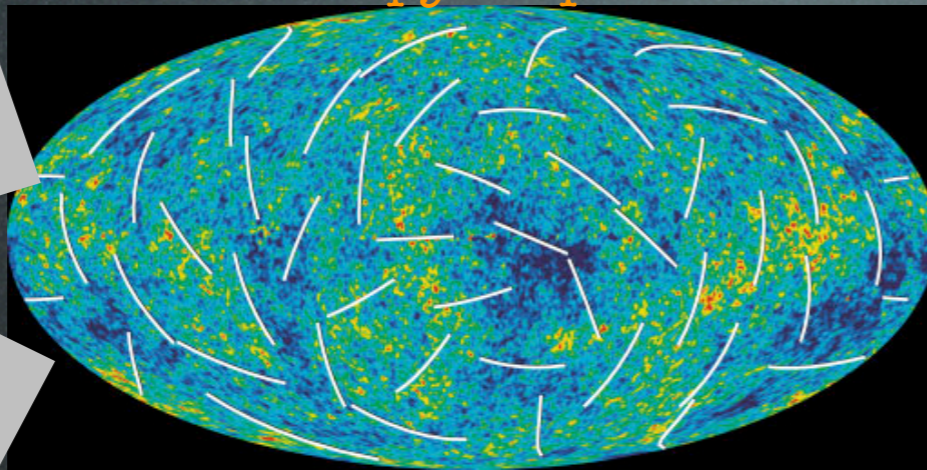
The dataset

( = some highly non trivial steps)

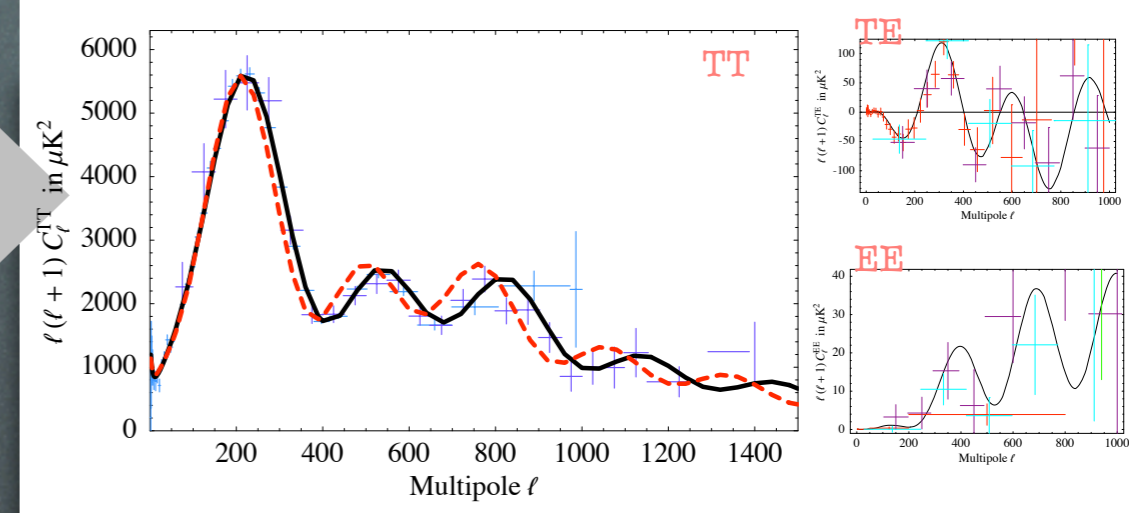
WMAP



CMB anisotropy map



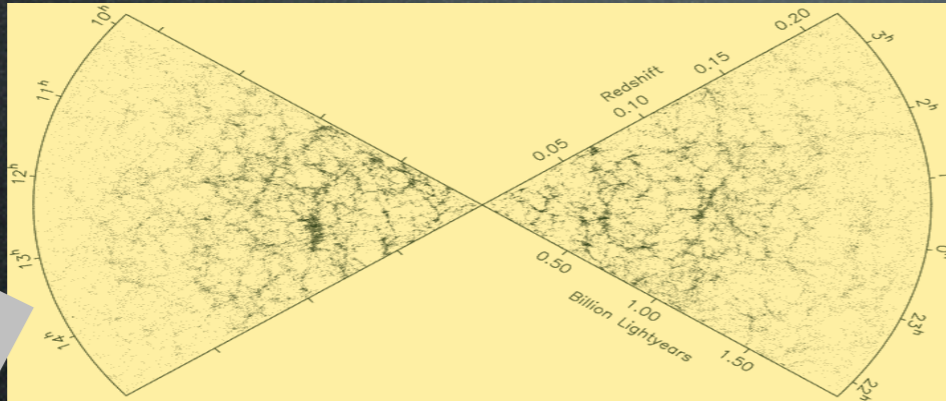
CMB power spectrum



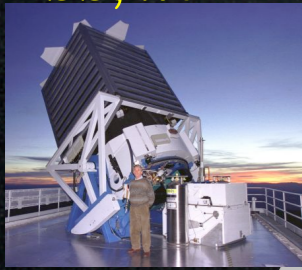
Boomerang...



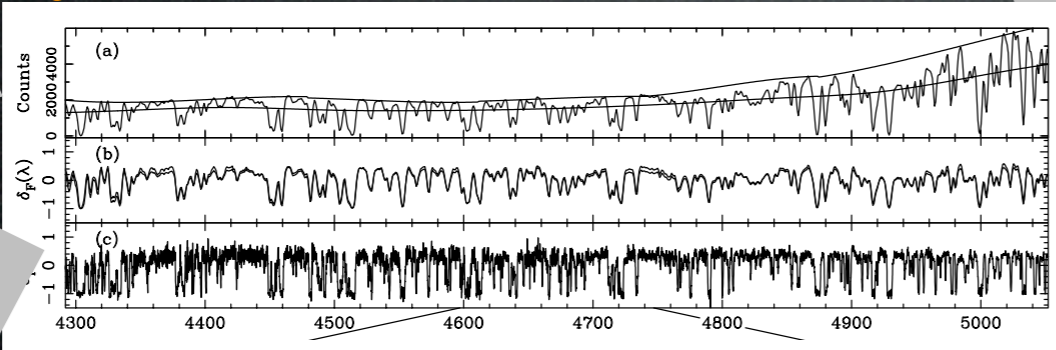
LSS redshift survey



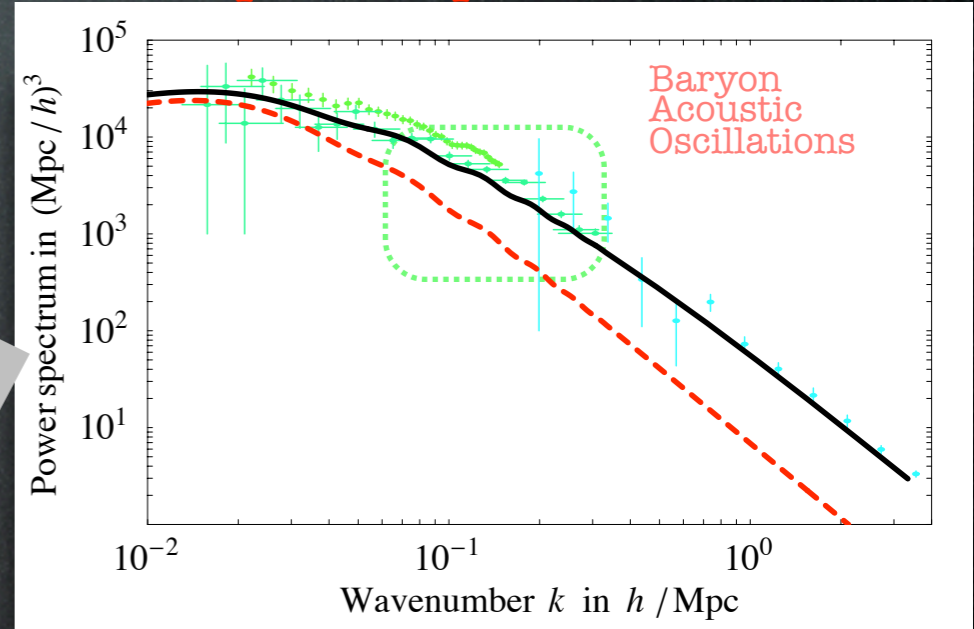
SDSS, 2dF



Lyman-alpha forest



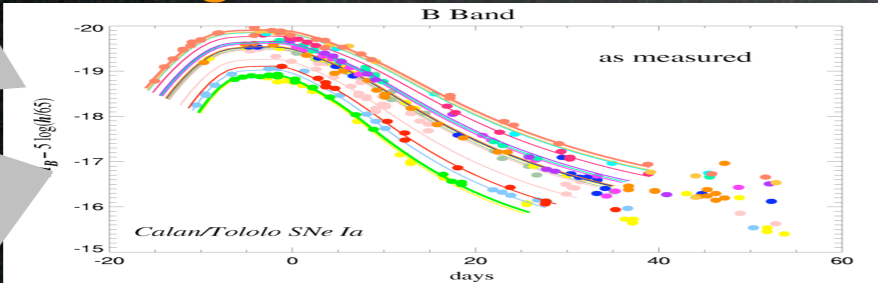
matter power spectrum



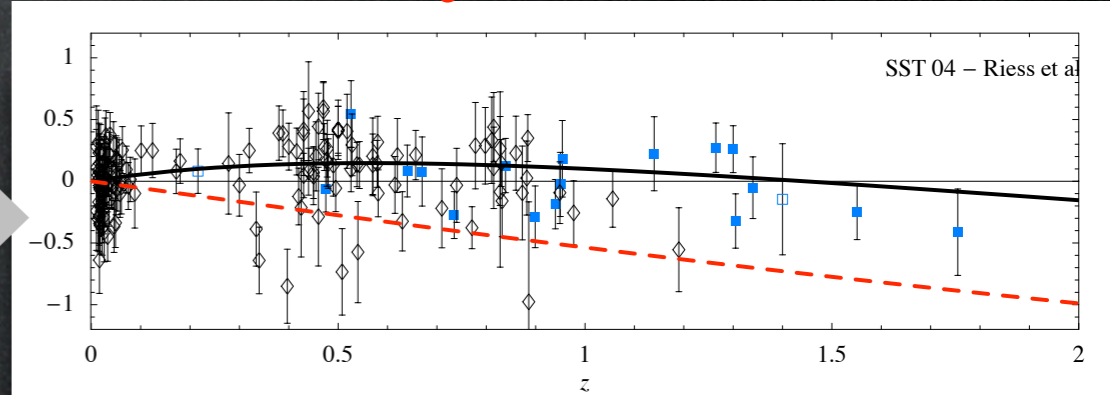
Keck, Hawaii



SNIa lightcurves



SNIa luminosity distance



HST



The dataset

CMB Temperature and Polarization:

- **WMAP 3-years** (TT, TE, EE spectra) [WMAP Science Team, astro-ph/0603449](#) →
- Boomerang 2003 (TT, TE, EE) [Boomerang Coll., astro-ph/0507494, astro-ph/0507507, astro-ph/0507514](#)
- ACBAR (TT) [Kuo et al., astro-ph/0212289](#) →
- CAPMAP (EE) [Barkats et al., astro-ph/0409380](#)
- CBI (TT, EE) [Readhead et al., astro-ph/0402359, astro-ph/0409569, Sievers et al., astro-ph/0509203](#) →
- DASI (TE, EE) [Leitch et al., astro-ph/0409357](#)
- VSA (TT) [Grainge et al., astro-ph/0212495](#)

LSS galaxy redshift surveys: dealing with bias and non-linearities as

- SDSS [SDSS Coll., astro-ph/0310725](#) →
- 2dF [2dF Coll., astro-ph/0501174](#)

$$P_{\text{gal}}(k) = b^2 \frac{1 + Q k^2}{1 + A k} P(k)$$

Baryon Acoustic Oscillations: in terms of a measurement of

[Eisenstein et al., astro-ph/0501171](#)

$$A = \left(\frac{D_A^2 c z}{H(z)} \right)^{1/3} \frac{\sqrt{\Omega_{\text{matter}} H_0^2}}{0.35 c}$$

Lyman- α Forest:

- Croft [Croft et al., astro-ph/0012324](#)
- SDSS [SDSS Coll., astro-ph/0407377](#)

Type Ia Supernovae:

- SST Gold sample [Riess et al., astro-ph/0402512](#) →
- SNLS [Astier et al., astro-ph/0510447](#) →


Hubble constant:

[HST Project, Freedman et al., astro-ph/0012376](#)

$$h = 0.72 \pm 0.08 \quad H_0 = 100h \text{ km/sec/Mpc}$$

The computational tool



We use **our own code** in  to as opposed to:

- evolve cosmological perturbations, CMBfast/CAMB
- compute spectra and CMBfast/CAMB
- run statistical comparisons with data. CosmoMC

Line-of-sight approach, Newtonian gauge.

Recombination is implemented calling *recfast*.

SZ background is marginalized over.

We adopt **gaussian** statistics.

MCMC

The computational tool



We use **our own code** in **MATHEMATICA 5.2** to **as opposed to:**

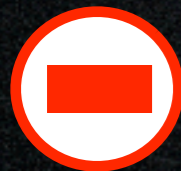
- evolve cosmological perturbations, **CMBfast/CAMB**
- compute spectra and **CMBfast/CAMB**
- run statistical comparisons with data. **CosmoMC**

Line-of-sight approach, Newtonian gauge.

Recombination is implemented calling *recfast*.

SZ background is marginalized over.

We adopt **gaussian** statistics. **MCMC**



slower, not fully optimized,
intrinsic gaussian “systematics”



customizable, analytic computations,
analytic dependance on cosmo parameters

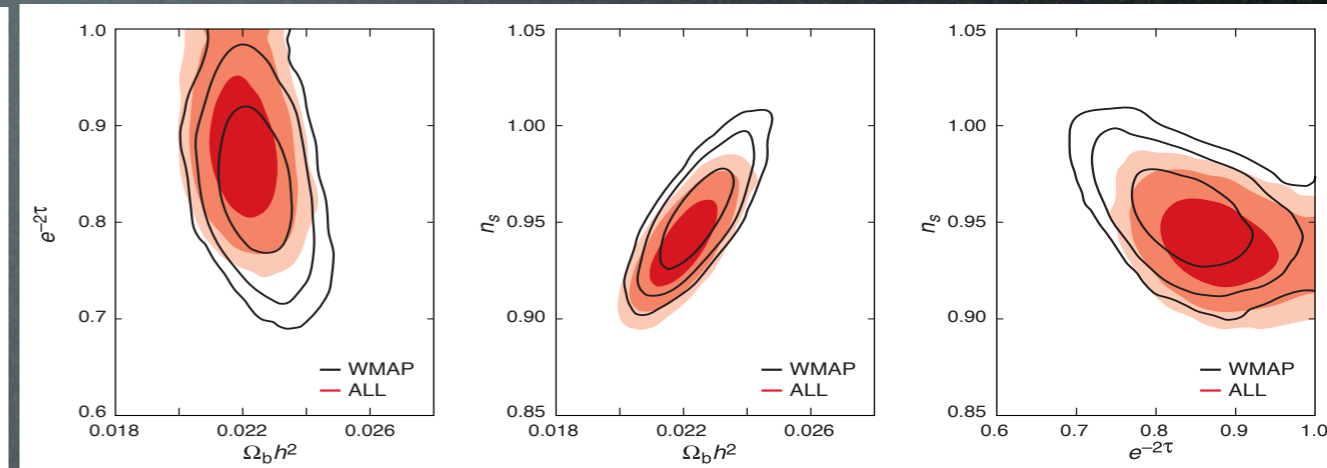
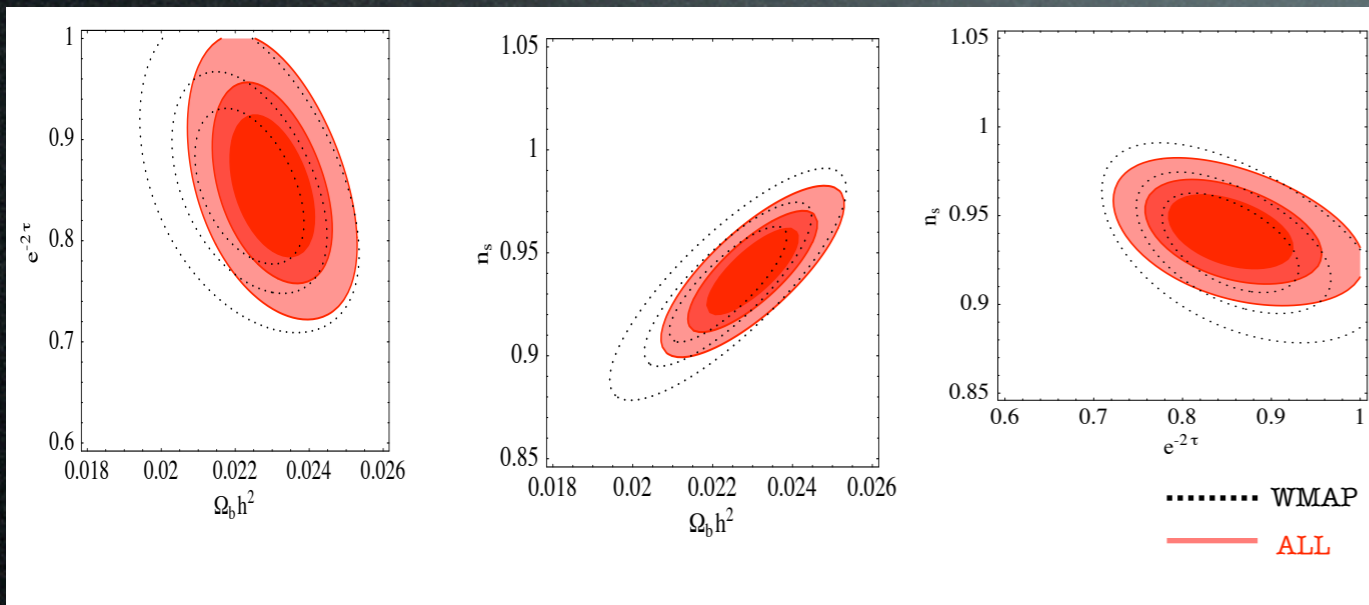
Comparing our code

[\[back\]](#)

Our analysis:



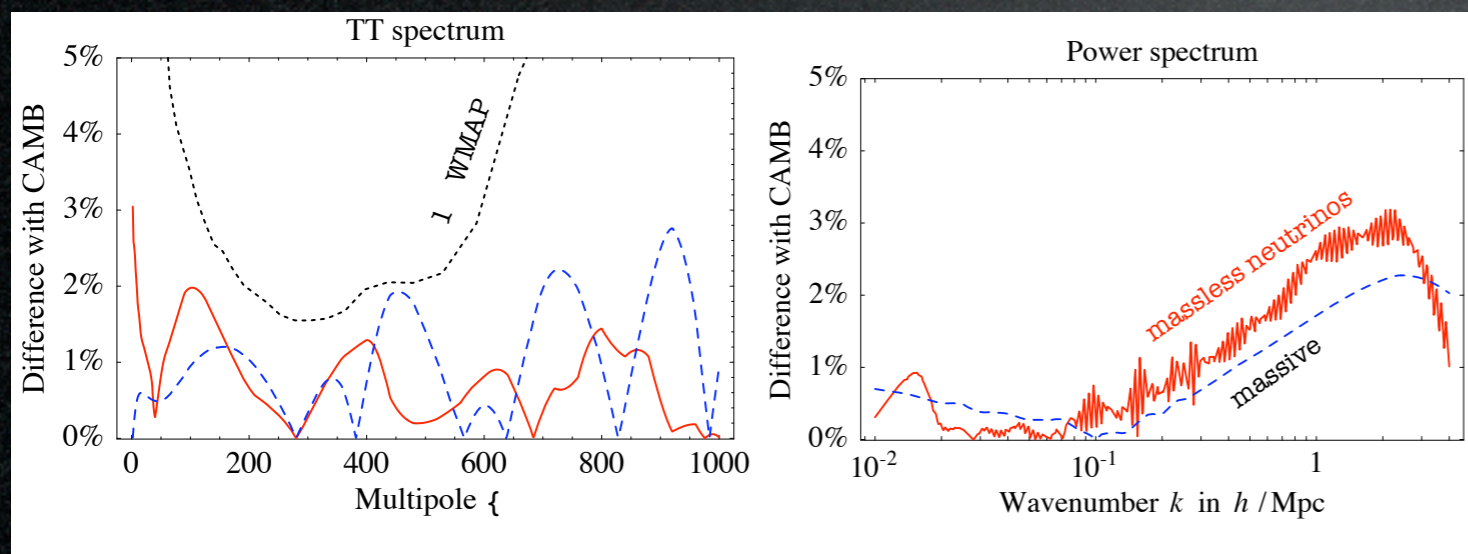
WMAP Science Team analysis:



[Spergel et al. WMAP 3yr results '05]


fit	A_s	h	n_s	τ	$100\Omega_b h^2$	$\Omega_{DM} h^2$
WMAP3	0.80 ± 0.05	0.704 ± 0.033	0.935 ± 0.019	0.081 ± 0.030	2.24 ± 0.10	0.113 ± 0.010
Global	0.84 ± 0.04	0.729 ± 0.013	0.951 ± 0.012	0.121 ± 0.025	2.36 ± 0.07	0.117 ± 0.003

Parameter	WMAP Only	WMAP+ SDSS	WMAP+ LRG	WMAP + SN Gold
$100\Omega_b h^2$	$2.233^{+0.072}_{-0.091}$	$2.233^{+0.062}_{-0.086}$	$2.242^{+0.062}_{-0.084}$	$2.227^{+0.065}_{-0.082}$
$\Omega_m h^2$	$0.1268^{+0.0073}_{-0.0128}$	$0.1329^{+0.0057}_{-0.0109}$	$0.1337^{+0.0047}_{-0.0098}$	$0.1349^{+0.0054}_{-0.0106}$
h	$0.734^{+0.028}_{-0.038}$	$0.709^{+0.024}_{-0.032}$	$0.709^{+0.016}_{-0.023}$	$0.701^{+0.020}_{-0.026}$
A	$0.801^{+0.043}_{-0.054}$	$0.813^{+0.042}_{-0.052}$	$0.816^{+0.042}_{-0.049}$	$0.827^{+0.045}_{-0.053}$
τ	$0.088^{+0.028}_{-0.034}$	$0.079^{+0.029}_{-0.032}$	$0.082^{+0.028}_{-0.033}$	$0.079^{+0.028}_{-0.034}$
n_s	$0.951^{+0.015}_{-0.019}$	$0.948^{+0.015}_{-0.018}$	$0.951^{+0.014}_{-0.018}$	$0.946^{+0.015}_{-0.019}$
σ_8	$0.744^{+0.050}_{-0.060}$	$0.772^{+0.036}_{-0.048}$	$0.781^{+0.032}_{-0.045}$	$0.784^{+0.035}_{-0.049}$
Ω_m	$0.238^{+0.027}_{-0.045}$	$0.266^{+0.025}_{-0.040}$	$0.267^{+0.017}_{-0.029}$	$0.276^{+0.022}_{-0.036}$




agreement is at **few %** level and within current precision of data

Results


(3 massive neutrinos)

How **heavy** are neutrinos?

Results

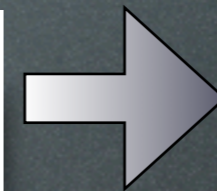
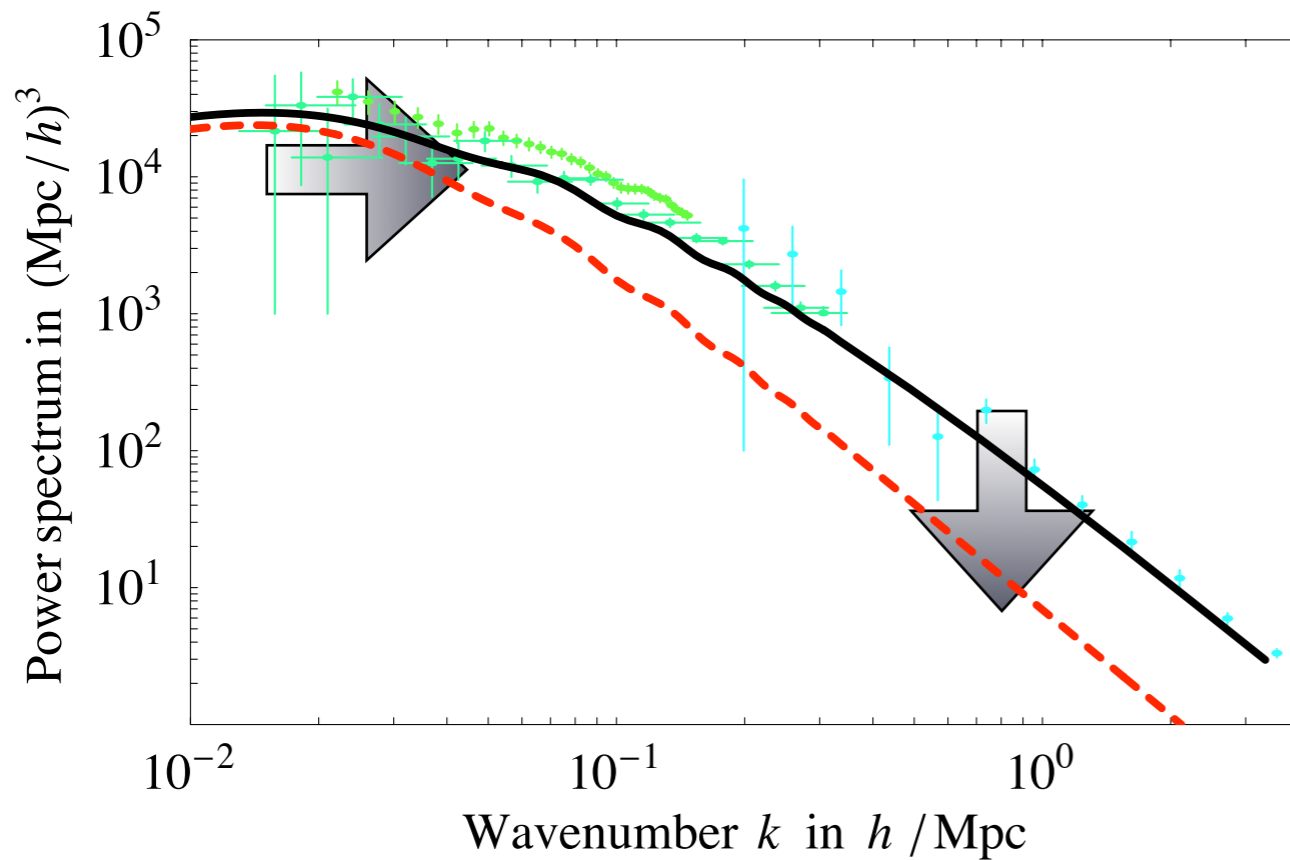

(3 massive neutrinos)

How **heavy** are neutrinos?

Cosmology probes $\sum m_{\nu_i}$.

— $m_\nu = 0$
- - - $m_\nu = 2 \text{ eV}$

Caveat: plots for illustrative purposes only,
all parameters except m_ν are held fixed.




$$k_{\text{NR}} = 0.018 \Omega_{\text{m}}^{-1/2} \left(\frac{m_\nu}{\text{eV}} \right)^{1/2} h_0 \text{ Mpc}^{-1}$$



$$\frac{\Delta P}{P} \simeq -8 f_\nu \quad \left(f_\nu = \frac{\sum m_{\nu_i} / 93 \text{ eV}}{\Omega_{\text{m}}} \right)$$

Results

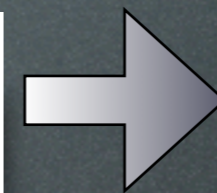
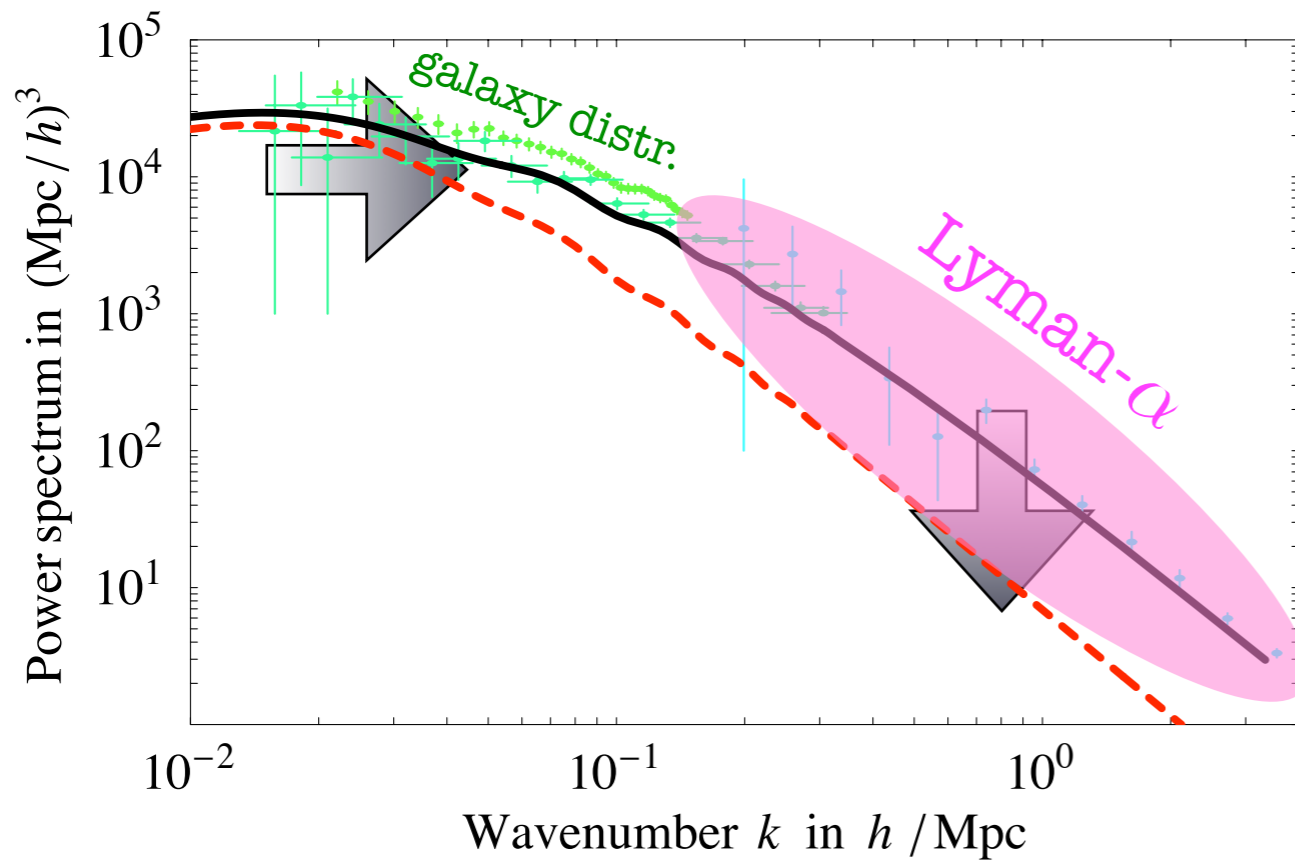

(3 massive neutrinos)

How **heavy** are neutrinos?

Cosmology probes $\sum m_{\nu_i}$.

— $m_\nu = 0$
- - - $m_\nu = 2 \text{ eV}$

Caveat: plots for illustrative purposes only,
all parameters except m_ν are held fixed.




$$k_{\text{NR}} = 0.018 \Omega_{\text{m}}^{-1/2} \left(\frac{m_\nu}{\text{eV}} \right)^{1/2} h_0 \text{ Mpc}^{-1}$$



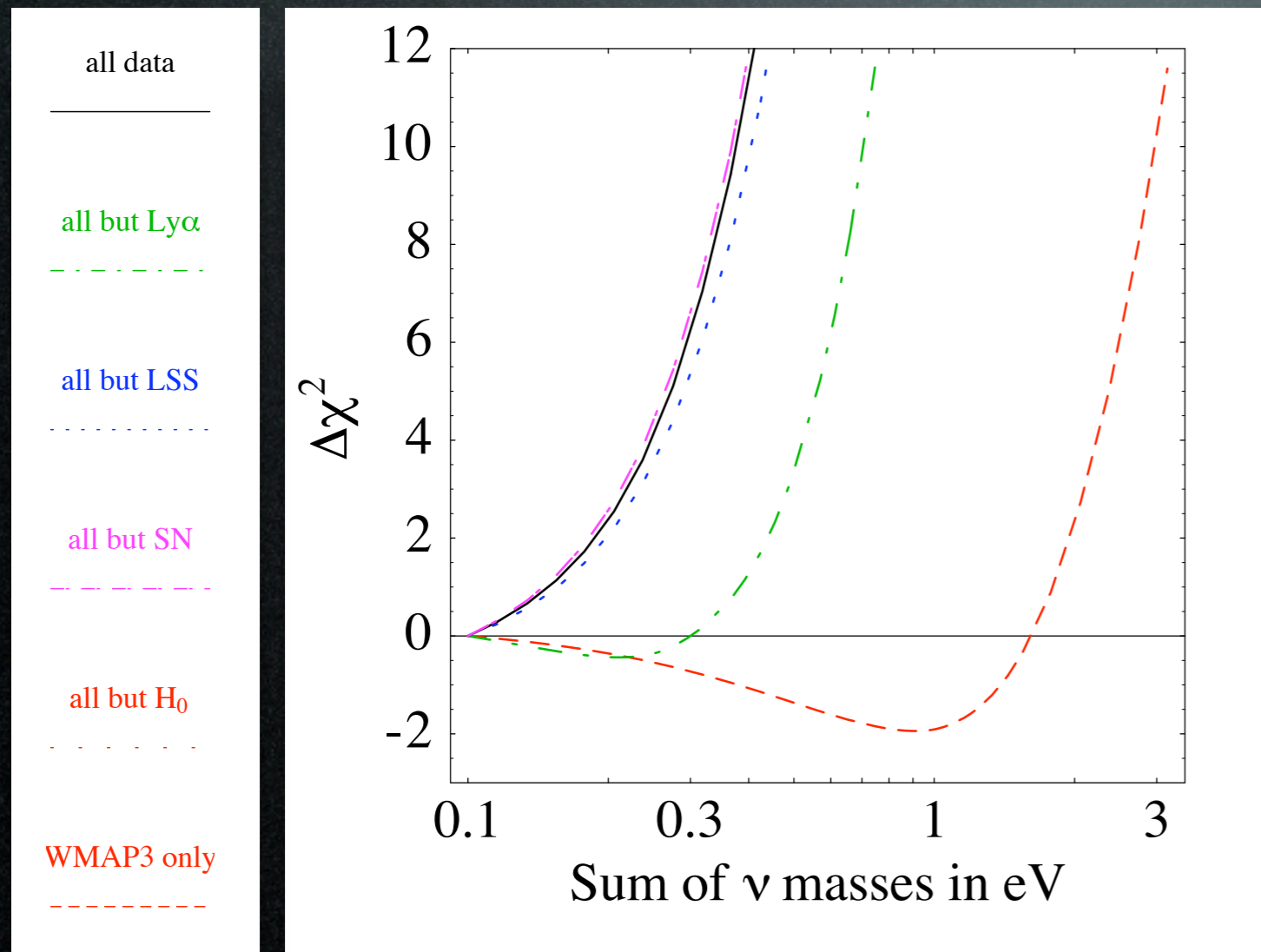
$$\frac{\Delta P}{P} \simeq -8 f_\nu \quad \left(f_\nu = \frac{\sum m_{\nu_i} / 93 \text{ eV}}{\Omega_{\text{m}}} \right)$$

Results


(3 massive neutrinos)

How **heavy** are neutrinos?

Cosmology probes $\sum m_{\nu_i}$.



CMB only:

$$\sum m_{\nu_i} < 2.2 \text{ eV} \quad (95\% \text{ C.L.})$$

Global fit:

$$\sum m_{\nu_i} < 0.40 \text{ eV} \quad (99.9\% \text{ C.L.})$$

dropping Ly-alpha:

$$\sum m_{\nu_i} < 0.73 \text{ eV} \quad (99.9\% \text{ C.L.})$$

Bottom Line:

Cosmology gives dominant bound on $\sum m_{\nu_i}$;
the bound tightens combining relatively less safe datasets.

Results

New neutrinos?



$3_{.04} \quad \Delta N_\nu$

Results

New neutrinos?

$$\underbrace{\text{red} \quad \text{green} \quad \text{blue}}_{3_{.04}} \quad \underbrace{\text{grey} \quad \text{grey}}_{\Delta N_\nu}$$

$$\left. \begin{aligned} \dot{\Theta} + ik\mu\Theta &= -\dot{\Phi} - ik\mu\Psi - \dot{\tau} \left[\Theta_0 - \Theta + \mu v_b - 1/2 \mathcal{P}_2(\mu)\Pi \right] \\ \dot{\Theta}_P + ik\mu\Theta_P &= -\dot{\tau} \left[\Theta_P + 1/2(1 - \mathcal{P}_2(\mu))\Pi \right] \end{aligned} \right\} \text{photons}$$

$\dot{\tau} = d\tau/d\eta = -n_e\sigma_T a \quad \Pi = \Theta_2 + \Theta_{P2} + \Theta_{P0}$

$$\left. \begin{aligned} \dot{\delta}_{\text{dm}} + ikv_{\text{dm}} &= -3\dot{\Phi} \\ \dot{v}_{\text{dm}} + \frac{\dot{a}}{a}v_{\text{dm}} &= -ik\Psi \end{aligned} \right\} \text{dark matter}$$

$$\left. \begin{aligned} \dot{\delta}_b + ikv_b &= -3\dot{\Phi} \\ \dot{v}_b + \frac{\dot{a}}{a}v_b &= -ik\Psi + \frac{\dot{\tau}}{R} \left[v_b + 3i\Theta_1 \right] \end{aligned} \right\} \text{baryons}$$

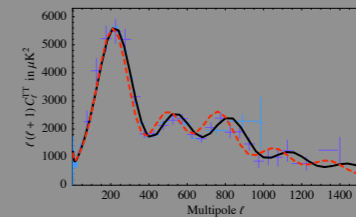
$R = 3\rho_b^0/4\rho_\gamma^0$

$$\left. \begin{aligned} \dot{\mathcal{N}} + i\frac{q_\nu}{E_\nu}k\mu\mathcal{N} &= -\dot{\Phi} - i\frac{E_\nu}{q_\nu}k\mu\Psi \end{aligned} \right\} \text{neutrinos}$$

$$\left. \begin{aligned} k^2\Phi + 3\frac{\dot{a}}{a}\left(\dot{\Phi} - \Psi\frac{\dot{a}}{a}\right) &= 4\pi G_N a^2 \left[\rho_m\delta_m + 4\rho_r\delta_r \right] \\ k^2(\Phi + \Psi) &= -32\pi G_N a^2 \rho_r \Theta_{r,2} \end{aligned} \right\} \text{metric}$$

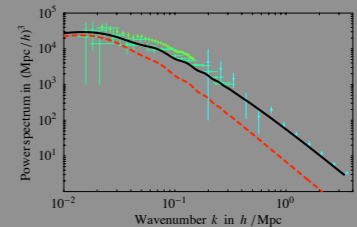
CMB Power spectrum

$$C_\ell \propto \int dk [\dots] \Theta_\ell(k)$$



Matter Power spect.

$$P(k) \propto \langle \delta_m(k)^2 \rangle$$



Results

New neutrinos?

$$\underbrace{\text{red} \text{ green} \text{ blue}}_{3_{.04}} \underbrace{\text{grey} \text{ grey}}_{\Delta N_\nu}$$

$$\left. \begin{aligned} \dot{\Theta} + ik\mu\Theta &= -\dot{\Phi} - ik\mu\Psi - \dot{\tau} \left[\Theta_0 - \Theta + \mu v_b - 1/2 \mathcal{P}_2(\mu)\Pi \right] \\ \dot{\Theta}_P + ik\mu\Theta_P &= -\dot{\tau} \left[\Theta_P + 1/2(1 - \mathcal{P}_2(\mu))\Pi \right] \end{aligned} \right\} \text{photons}$$

$\dot{\tau} = d\tau/d\eta = -n_e\sigma_T a \quad \Pi = \Theta_{-2} + \Theta_{P2} + \Theta_{P0}$

$$\left. \begin{aligned} \dot{\delta}_{\text{dm}} + ikv_{\text{dm}} &= -3\dot{\Phi} \\ \dot{v}_{\text{dm}} + \frac{\dot{a}}{a}v_{\text{dm}} &= -ik\Psi \end{aligned} \right\} \text{dark matter}$$

$$\left. \begin{aligned} \dot{\delta}_b + ikv_b &= -3\dot{\Phi} \\ \dot{v}_b + \frac{\dot{a}}{a}v_b &= -ik\Psi + \frac{\dot{\tau}}{R} [v_b + 3i\Theta_1] \end{aligned} \right\} \text{baryons}$$

$R = 3\rho_b^0/4\rho_\gamma^0$

$$\dot{\mathcal{N}} + i\frac{q_\nu}{E_\nu}k\mu\mathcal{N} = -\dot{\Phi} - i\frac{E_\nu}{q_\nu}k\mu\Psi \quad \text{neutrinos}$$

all N_ν rel degrees of freedom contribute to the energy density

$$\rho_r = \rho_\gamma \left[1 + \frac{7}{8}N_\nu \left(\frac{T_\nu}{T} \right)^4 \right]$$

with $\frac{T_\nu}{T} = \left(\frac{4}{11} \right)^{1/3}$

$$\left. \begin{aligned} k^2\Phi + 3\frac{\dot{a}}{a}\left(\dot{\Phi} - \Psi\frac{\dot{a}}{a}\right) &= 4\pi G_N a^2 [\rho_m\delta_m + 4\rho_r\delta_r] \\ k^2(\Phi + \Psi) &= -32\pi G_N a^2 \rho_r \Theta_{r,2} \end{aligned} \right\} \text{metric}$$

CMB Power spectrum

$$C_\ell \propto [dk[\dots]] \Theta_\ell(k)$$

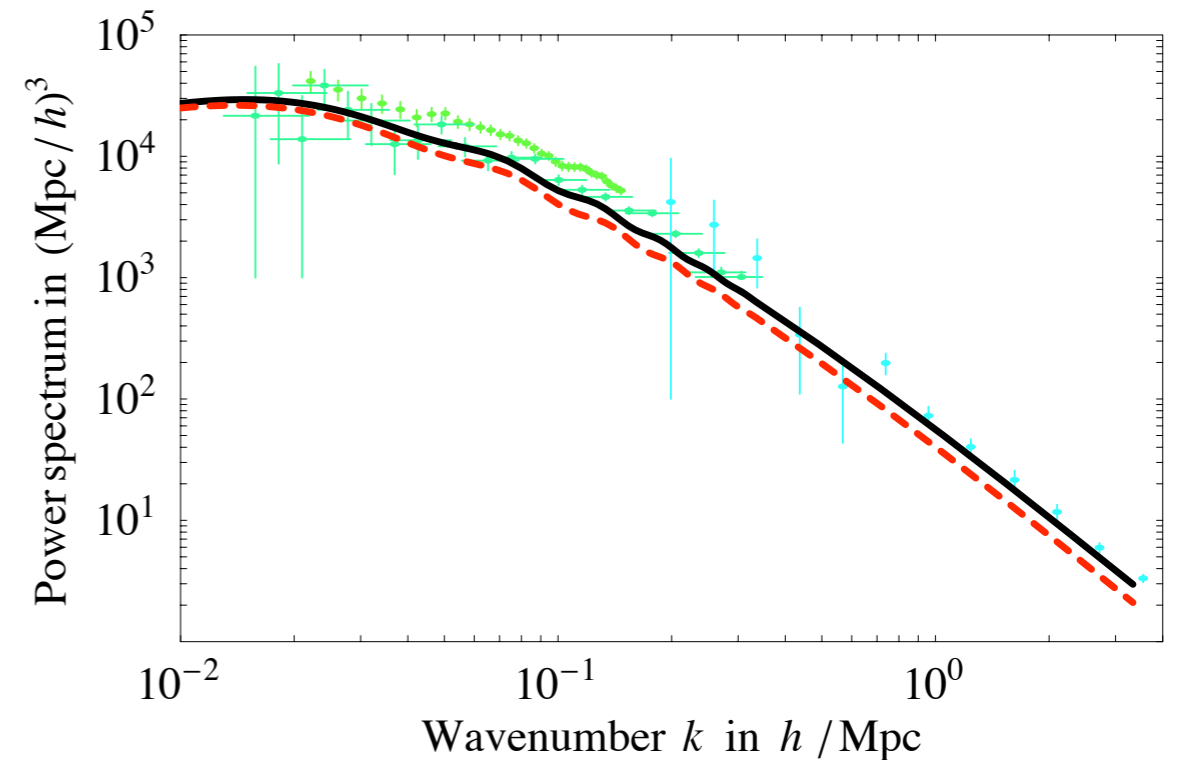
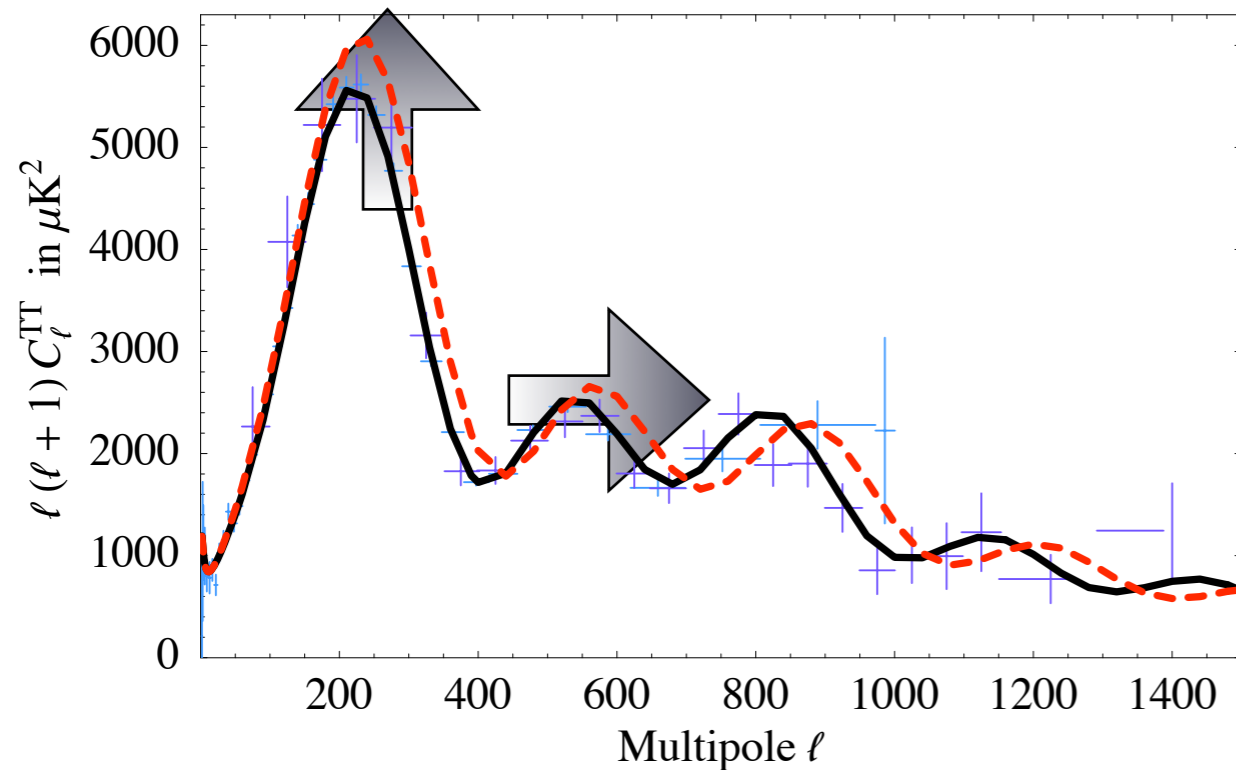
Results

New neutrinos?

$$\underbrace{\text{red circle} \quad \text{green circle} \quad \text{blue circle}}_{3.04} \quad \underbrace{\text{grey circle} \quad \text{white circle}}_{\Delta N_\nu}$$

— $N_\nu = 3$
- - - $N_\nu = 5$

Caveat: plots for illustrative purposes only,
all parameters except N_ν are held fixed
(here this caveat is particularly important).

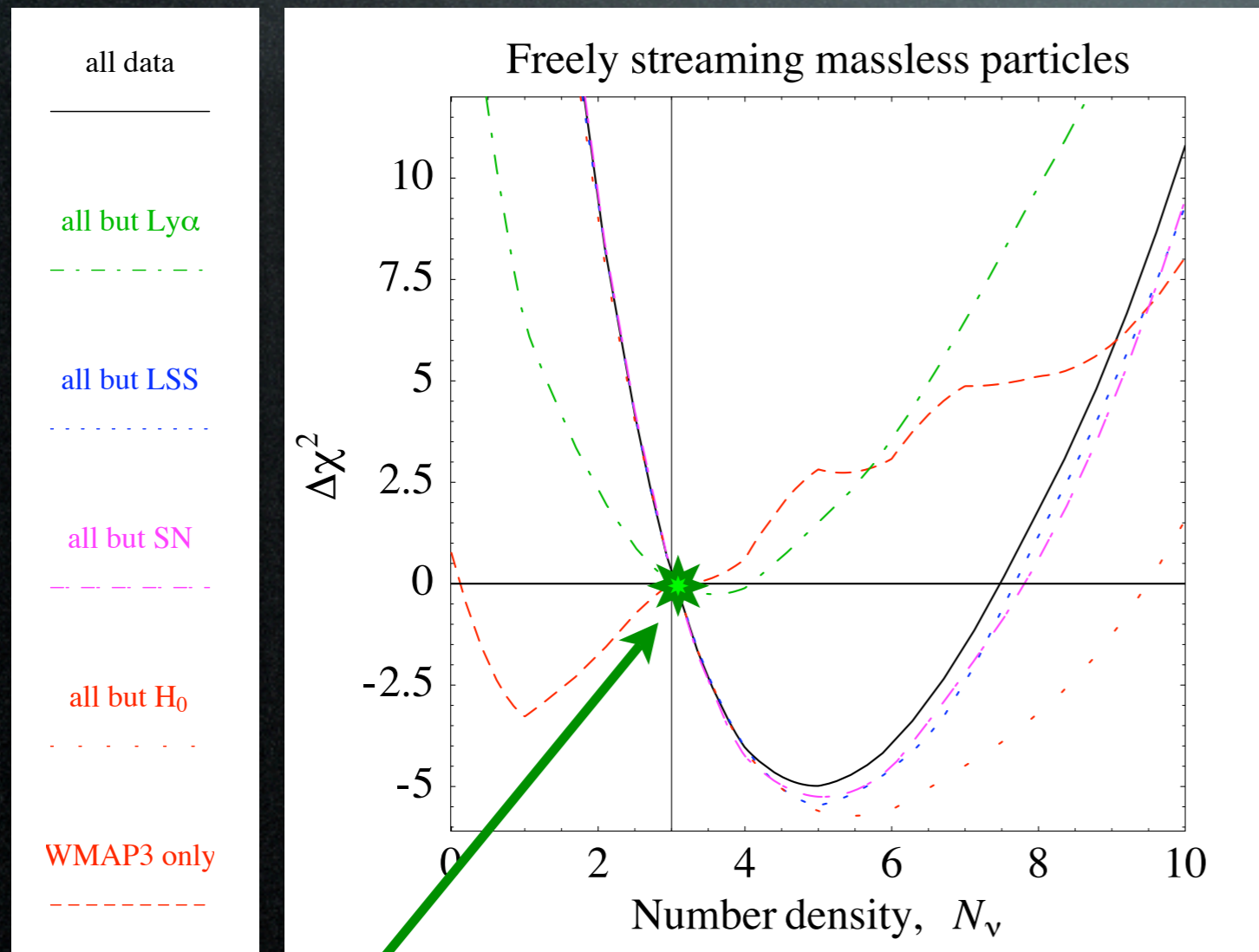


Results

New neutrinos?

$$\underbrace{\text{red circle} \text{ green circle} \text{ blue circle}}_{3.04} \underbrace{\text{grey circle} \text{ grey circle}}_{\Delta N_\nu}$$

All N_ν relativistic degrees of freedom contribute to the energy density.



Global fit:

$$N_\nu = 5 \pm 1$$

Cirelli, Strumia 2006

Seljak et al. 2006

Mangano et al. 2006

...

dropping Ly-alpha gives back

$$N_\nu \simeq 3$$

Cirelli, Strumia 2006

Standard cosmology

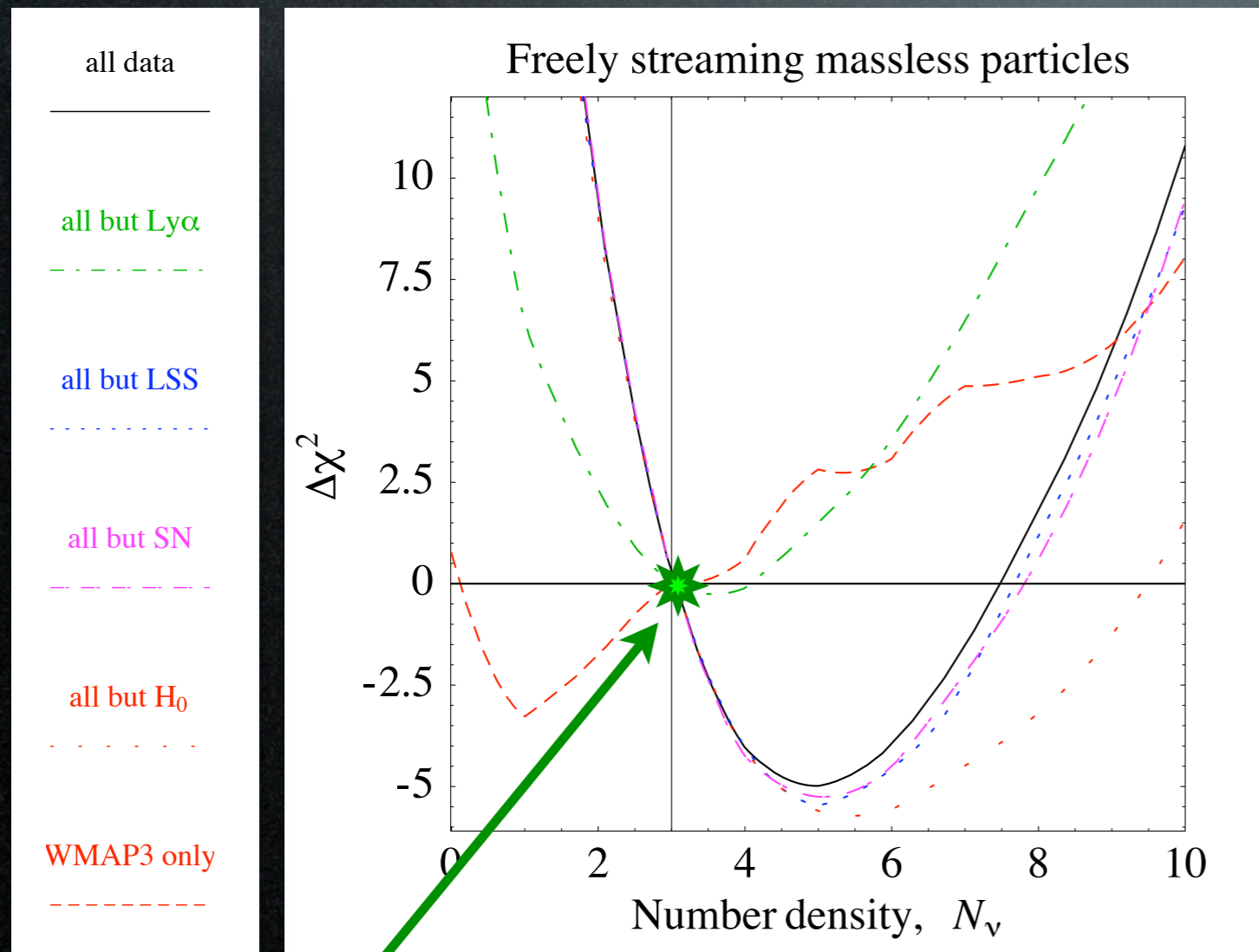
Bottom Line: Cosmology seems to suggest **5 neutrinos** (2 extra);
but Ly-alpha are mainly driving the suggestion.

Results

New neutrinos?

$$\underbrace{\text{red circle} \text{ green circle} \text{ blue circle}}_{3.04} \underbrace{\text{grey circle} \text{ grey circle}}_{\Delta N_\nu}$$

All N_ν relativistic degrees of freedom contribute to the energy density.



Standard cosmology

Global fit:

$$N_\nu = 5 \pm 1$$

Cirelli, Strumia 2006

Seljak et al. 2006

Mangano et al. 2006

...

dropping Ly-alpha gives back

$$N_\nu \simeq 3$$

Cirelli, Strumia 2006



just systematics?

see Hamann, Hannestad, Raffelt, Wong 2007

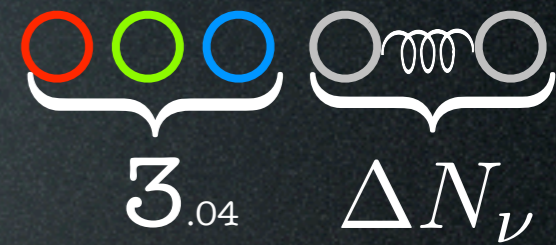
tension with BBN?

see Ichikawa et al. 2007

Bottom Line: Cosmology seems to suggest **5 neutrinos** (2 extra);
but Ly-alpha are mainly driving the suggestion.

Results

New sticky particles?



The diagram shows two Feynman diagrams for neutrino mass generation. The left diagram consists of three circles (red, green, blue) connected by a horizontal line, with a curly bracket underneath labeled $3_{.04}$. The right diagram consists of two circles connected by a horizontal line with a wavy line in the middle, with a curly bracket underneath labeled ΔN_ν .

$$3_{.04} \quad \Delta N_\nu$$

Results

$$\underbrace{\text{red circle} \text{ green circle} \text{ blue circle}}_{3_{.04}} \quad \underbrace{\text{grey circle} \text{ wavy line} \text{ grey circle}}_{\Delta N_\nu}$$

New **sticky** particles?

$$\left. \begin{aligned} \dot{\Theta} + ik\mu\Theta &= -\dot{\Phi} - ik\mu\Psi - \dot{\tau} \left[\Theta_0 - \Theta + \mu v_b - 1/2 \mathcal{P}_2(\mu)\Pi \right] \\ \dot{\Theta}_P + ik\mu\Theta_P &= -\dot{\tau} \left[\Theta_P + 1/2(1 - \mathcal{P}_2(\mu))\Pi \right] \end{aligned} \right\} \begin{array}{l} \dot{\tau} = d\tau/d\eta = -n_e\sigma_T a \quad \Pi = \Theta_{-2} + \Theta_{P2} + \Theta_{P0} \\ \text{photons} \end{array}$$

$$\left. \begin{aligned} \dot{\delta}_{\text{dm}} + ikv_{\text{dm}} &= -3\dot{\Phi} \\ \dot{v}_{\text{dm}} + \frac{\dot{a}}{a}v_{\text{dm}} &= -ik\Psi \end{aligned} \right\} \text{dark matter}$$

$$\left. \begin{aligned} \dot{\delta}_b + ikv_b &= -3\dot{\Phi} \\ \dot{v}_b + \frac{\dot{a}}{a}v_b &= -ik\Psi + \frac{\dot{\tau}}{R} \left[v_b + 3i\Theta_1 \right] \end{aligned} \right\} \begin{array}{l} R = 3\rho_b^0/4\rho_\gamma^0 \\ \text{baryons} \end{array}$$

$$\left. \dot{\mathcal{N}} + i\frac{q_\nu}{E_\nu}k\mu\mathcal{N} = -\dot{\Phi} - i\frac{E_\nu}{q_\nu}k\mu\Psi \right\} \text{neutrinos}$$

$$\left. \begin{aligned} \dot{\delta}_x + i\frac{4}{3}kv_x &= -4\dot{\Phi} \\ \dot{v}_x + \frac{i}{4}k\delta_x &= -ik\Psi \end{aligned} \right\} \text{extra}$$

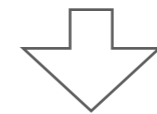
$$\left. \begin{aligned} k^2\Phi + 3\frac{\dot{a}}{a}\left(\dot{\Phi} - \Psi\frac{\dot{a}}{a}\right) &= 4\pi G_N a^2 [\rho_m\delta_m + 4\rho_r\delta_r] \\ k^2(\Phi + \Psi) &= -32\pi G_N a^2 \rho_r \Theta_{r,2} \end{aligned} \right\} \text{metric}$$

Massless particles, interacting among themselves (i.e. **non freely streaming**) at the time of CMB formation.

e.g. a scalar φ with $\lambda\varphi^4$

e.g. scalar + fermion with $\lambda'\varphi\nu_s^2$

e.g. fermions with $\langle\bar{N}N\rangle$...



A relativistic fluid: δ_x, v_x .

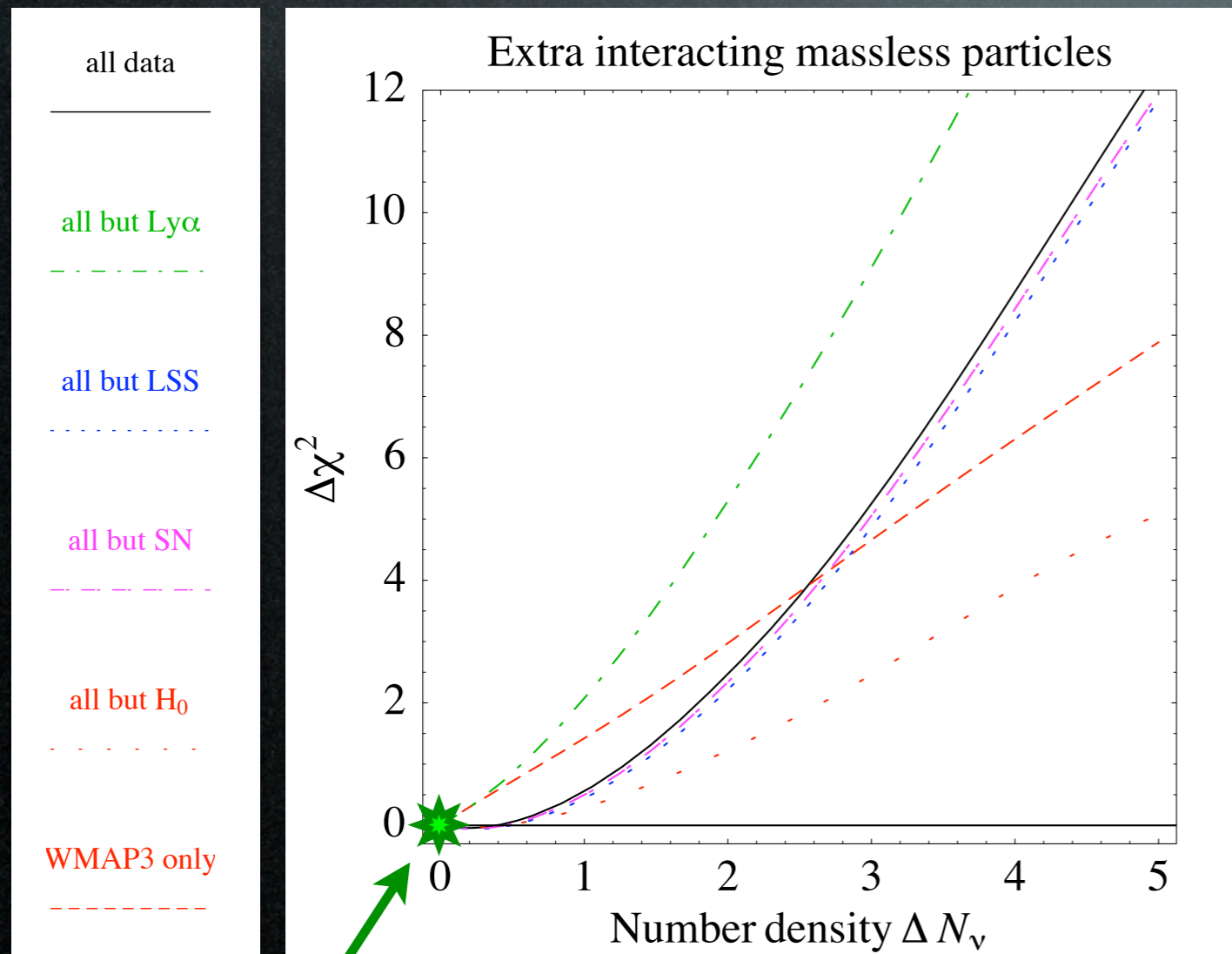
Contributes $\Delta N_\nu \cdot \delta_x$ to the rel energy density.

Results

New **sticky** particles?

$\underbrace{\text{red circle} \text{ green circle} \text{ blue circle}}_{3.04} \quad \underbrace{\text{grey circle} \text{ wavy line} \text{ grey circle}}_{\Delta N_\nu}$

ΔN_ν extra massless particles interacting among themselves.



Global fit:

$$\Delta N_\nu = 0 \pm 1.3$$

Standard cosmology

Bottom Line: Cosmology **constrains**
extra massless sticky particles.

[skip massive ν]

Results

New massive neutrinos?



Results

$$\underbrace{\text{red circle} \quad \text{green circle} \quad \text{blue circle} \quad \text{grey circle} \quad \text{grey circle}}_{\Delta N_\nu, m_s}$$

New **massive** neutrinos?

$$\left. \begin{aligned} \dot{\Theta} + ik\mu\Theta &= -\dot{\Phi} - ik\mu\Psi - \dot{\tau} \left[\Theta_0 - \Theta + \mu v_b - 1/2 \mathcal{P}_2(\mu)\Pi \right] \\ \dot{\Theta}_P + ik\mu\Theta_P &= -\dot{\tau} \left[\Theta_P + 1/2(1 - \mathcal{P}_2(\mu))\Pi \right] \end{aligned} \right\} \text{photons}$$

$\dot{\tau} = d\tau/d\eta = -n_e\sigma_T a \quad \Pi = \Theta_{-2} + \Theta_{P2} + \Theta_{P0}$

$$\left. \begin{aligned} \dot{\delta}_{\text{dm}} + ikv_{\text{dm}} &= -3\dot{\Phi} \\ \dot{v}_{\text{dm}} + \frac{\dot{a}}{a}v_{\text{dm}} &= -ik\Psi \end{aligned} \right\} \text{dark matter}$$

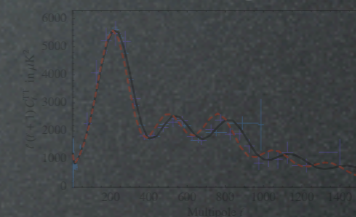
$$\left. \begin{aligned} \dot{\delta}_b + ikv_b &= -3\dot{\Phi} \\ \dot{v}_b + \frac{\dot{a}}{a}v_b &= -ik\Psi + \frac{\dot{\tau}}{R} [v_b + 3i\Theta_1] \end{aligned} \right\} \text{baryons}$$

$R = 3\rho_b^0/4\rho_\gamma^0$

$$\left. \dot{\mathcal{N}} + i\frac{q_\nu}{E_\nu}k\mu\mathcal{N} = -\dot{\Phi} - i\frac{E_\nu}{q_\nu}k\mu\Psi \right\} \text{neutrinos}$$

CMB Power spectrum

$$C_\ell \propto \int dk [\dots] \Theta_\ell(k)$$



3 standard neutrinos
+

ΔN_ν neutrinos with mass m_s .

Contribute to the Rel/NR energy densities.

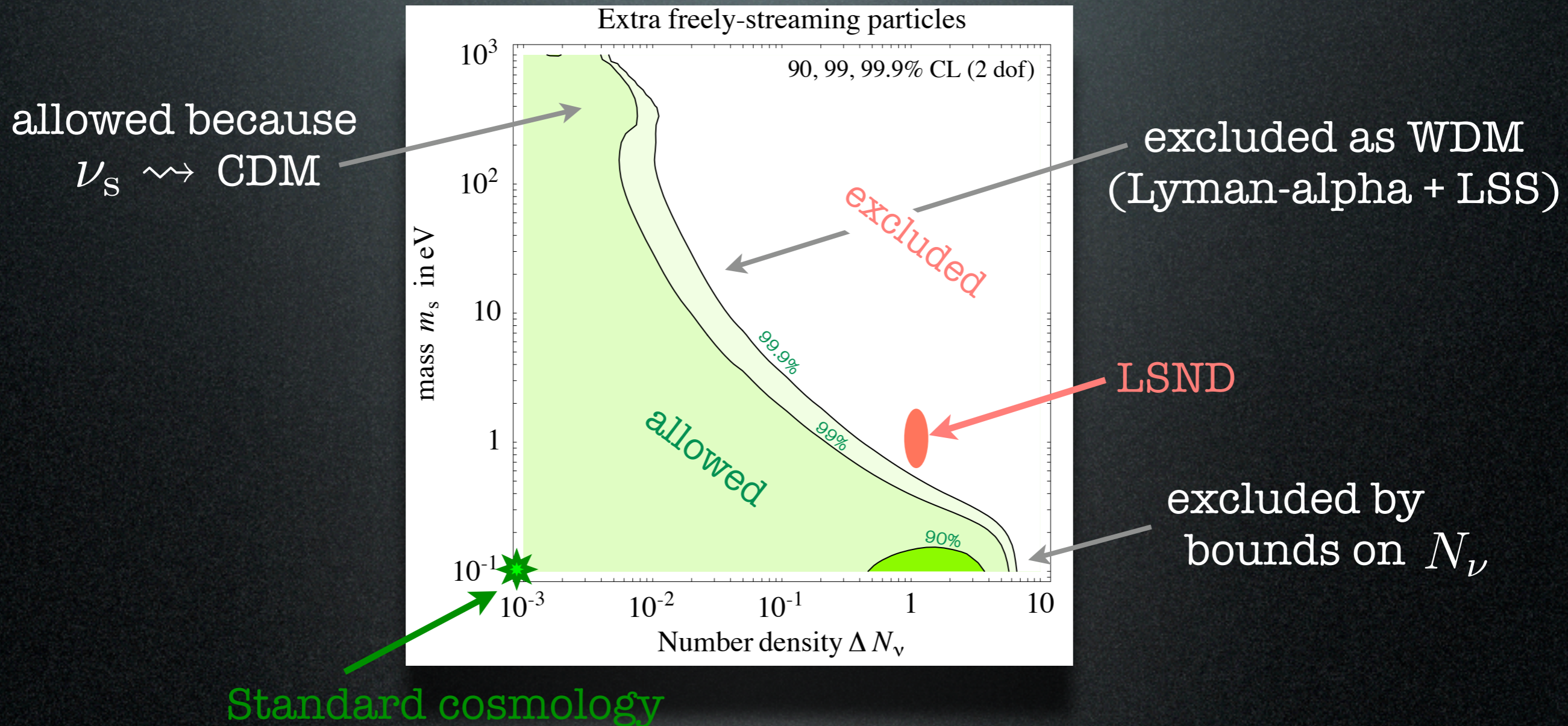
$$\left. \begin{aligned} k^2\Phi + 3\frac{\dot{a}}{a}\left(\dot{\Phi} - \Psi\frac{\dot{a}}{a}\right) &= 4\pi G_N a^2 [\rho_m\delta_m + 4\rho_r\delta_r] \\ k^2(\Phi + \Psi) &= -32\pi G_N a^2 \rho_r \Theta_{r,2} \end{aligned} \right\} \text{metric}$$

Results

New massive neutrinos?

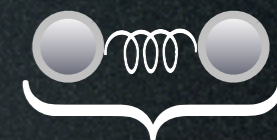
$\Delta N_\nu, m_s$

3 standard neutrinos + ΔN_ν neutrinos with mass m_s .



Results

New massive, sticky particles?



$\Delta N_\nu, m_s$

Results



New massive, sticky particles?

$$\left. \begin{aligned} \dot{\Theta} + ik\mu\Theta &= -\dot{\Phi} - ik\mu\Psi - \dot{\tau} \left[\Theta_0 - \Theta + \mu v_b - 1/2 \mathcal{P}_2(\mu)\Pi \right] \\ \dot{\Theta}_P + ik\mu\Theta_P &= -\dot{\tau} \left[\Theta_P + 1/2(1 - \mathcal{P}_2(\mu))\Pi \right] \end{aligned} \right\} \text{photons}$$

$\dot{\tau} = d\tau/d\eta = -n_e\sigma_T a \quad \Pi = \Theta_{-2} + \Theta_{P2} + \Theta_{P0}$

$$\left. \begin{aligned} \dot{\delta}_{\text{dm}} + ikv_{\text{dm}} &= -3\dot{\Phi} \\ \dot{v}_{\text{dm}} + \frac{\dot{a}}{a}v_{\text{dm}} &= -ik\Psi \end{aligned} \right\} \text{dark matter}$$

$$\left. \begin{aligned} \dot{\delta}_b + ikv_b &= -3\dot{\Phi} \\ \dot{v}_b + \frac{\dot{a}}{a}v_b &= -ik\Psi + \frac{\dot{\tau}}{R} \left[v_b + 3i\Theta_1 \right] \end{aligned} \right\} \text{baryons}$$

$R = 3\rho_b^0/4\rho_\gamma^0$

$$\left. \dot{\mathcal{N}} + i\frac{q_\nu}{E_\nu}k\mu\mathcal{N} = -\dot{\Phi} - i\frac{E_\nu}{q_\nu}k\mu\Psi \right\} \text{neutrinos}$$

$$\left. \begin{aligned} \dot{\delta}_x &= -(1+w)(3\dot{\Phi} + ikv_x) \\ \dot{v}_x &= -ik\Psi + \frac{\dot{a}}{a}(1-3w)iv_x - \frac{w}{1+w}ik\delta_x \end{aligned} \right\} \text{extra}$$

$$\left. \begin{aligned} k^2\Phi + 3\frac{\dot{a}}{a}\left(\dot{\Phi} - \Psi\frac{\dot{a}}{a}\right) &= 4\pi G_N a^2 [\rho_m\delta_m + 4\rho_r\delta_r] \\ k^2(\Phi + \Psi) &= -32\pi G_N a^2 \rho_r \Theta_{r,2} \end{aligned} \right\} \text{metric}$$

CMB Power spectrum

Massive particles,
interacting among themselves
(i.e. non freely streaming).



A fluid defined by δ_x, v_x ,
with $w = 1/3$ when rel,
 $w = 0$ when NR.

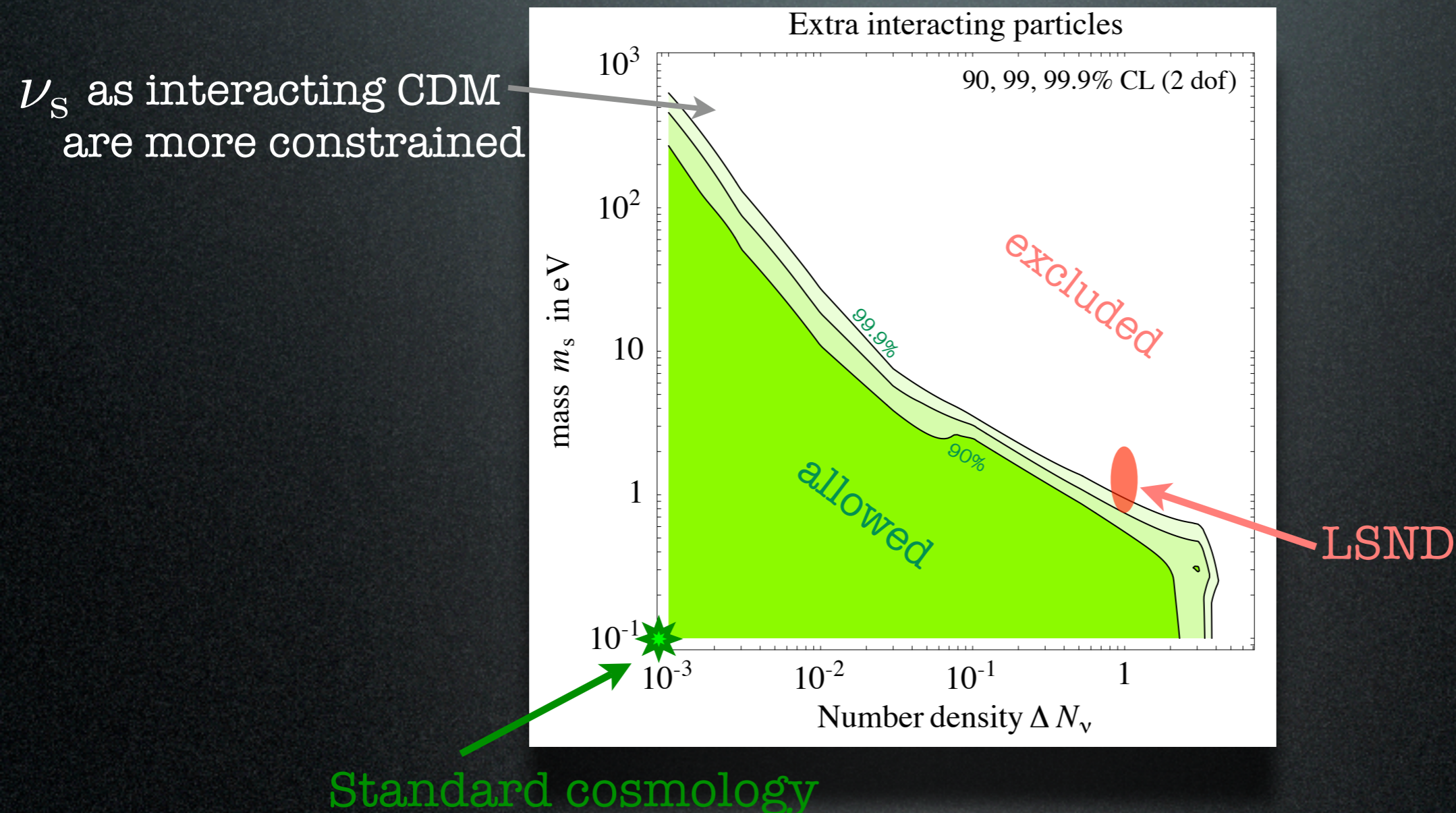
Contribute to the Rel/NR
energy densities.

Results

New massive, sticky particles?



3 standard neutrinos + ΔN_ν with mass m_s , interacting among themselves



Bottom Line: Cosmology **constrains** extra sterile neutrinos (freely-streaming or interacting): they better be few and light.

Results

Can some neutrinos be **sticky**?

$$N_{\nu}^{\text{norm}} \begin{cases} \text{red circle} \\ \text{green circle} \end{cases} \\ N_{\nu}^{\text{int}} \begin{cases} \text{blue circle} \\ \text{grey circle} \end{cases} \begin{cases} \text{wavy line} \\ \text{wavy line} \end{cases} \begin{cases} \text{square} \\ \text{square} \end{cases} \Big\} N_{\phi}$$

Results

Can some neutrinos be **sticky**?

$$\begin{aligned}\dot{\Theta} + ik\mu\Theta &= -\dot{\Phi} - ik\mu\Psi - \dot{\tau}[\Theta_0 - \Theta + \mu v_b - 1/2 \mathcal{P}_2(\mu)\Pi] \\ \dot{\Theta}_P + ik\mu\Theta_P &= -\dot{\tau}[\Theta_P + 1/2(1 - \mathcal{P}_2(\mu))\Pi]\end{aligned}$$

$$\dot{\tau} = d\tau/d\eta = -n_e\sigma_T a \quad \Pi = \Theta_2 + \Theta_{P2} + \Theta_{P0}$$

$$N_\nu^{\text{norm}} \left\{ \begin{array}{l} \text{red circle} \\ \text{green circle} \end{array} \right\} N_\nu^{\text{int}} \left\{ \begin{array}{l} \text{blue circle} \\ \text{grey circle} \end{array} \right\} N_\phi \left\{ \begin{array}{l} \text{wavy line} \\ \text{square} \end{array} \right\}$$

photons

$$\left. \begin{aligned}\dot{\delta}_{\text{dm}} + ikv_{\text{dm}} &= -3\dot{\Phi} \\ \dot{v}_{\text{dm}} + \frac{\dot{a}}{a}v_{\text{dm}} &= -ik\Psi\end{aligned} \right\} \text{dark matter}$$

$$\left. \begin{aligned}\dot{\delta}_b + ikv_b &= -3\dot{\Phi} \\ \dot{v}_b + \frac{\dot{a}}{a}v_b &= -ik\Psi + \frac{\dot{\tau}}{R}[v_b + 3i\Theta_1]\end{aligned} \right\} \text{baryons}$$

$$R = 3\rho_b^0/4\rho_\gamma^0$$

$$\left. \begin{aligned}\dot{\mathcal{N}} + i\frac{q_\nu}{E_\nu}k\mu\mathcal{N} &= -\dot{\Phi} - i\frac{E_\nu}{q_\nu}k\mu\Psi\end{aligned} \right\} \text{neutrinos}$$

$$\left. \begin{aligned}\dot{\delta}_x + i\frac{4}{3}kv_x &= -4\dot{\Phi} \\ \dot{v}_x + \frac{i}{4}k\delta_x &= -ik\Psi\end{aligned} \right\} \text{extra}$$

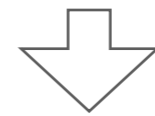
$$\begin{aligned}k^2\Phi + 3\frac{\dot{a}}{a}\left(\dot{\Phi} - \Psi\frac{\dot{a}}{a}\right) &= 4\pi G_N a^2[\rho_m\delta_m + 4\rho_r\delta_r] \\ k^2(\Phi + \Psi) &= -32\pi G_N a^2\rho_r\Theta_{r,2}\end{aligned}$$

metric

Neutrinos interacting with extra particles such that free-streaming is prevented.

e.g. effective couplings $g\nu\nu\varphi$

$$g'\nu\nu_s\varphi$$



N_ν^{norm} freely-streaming neutrinos

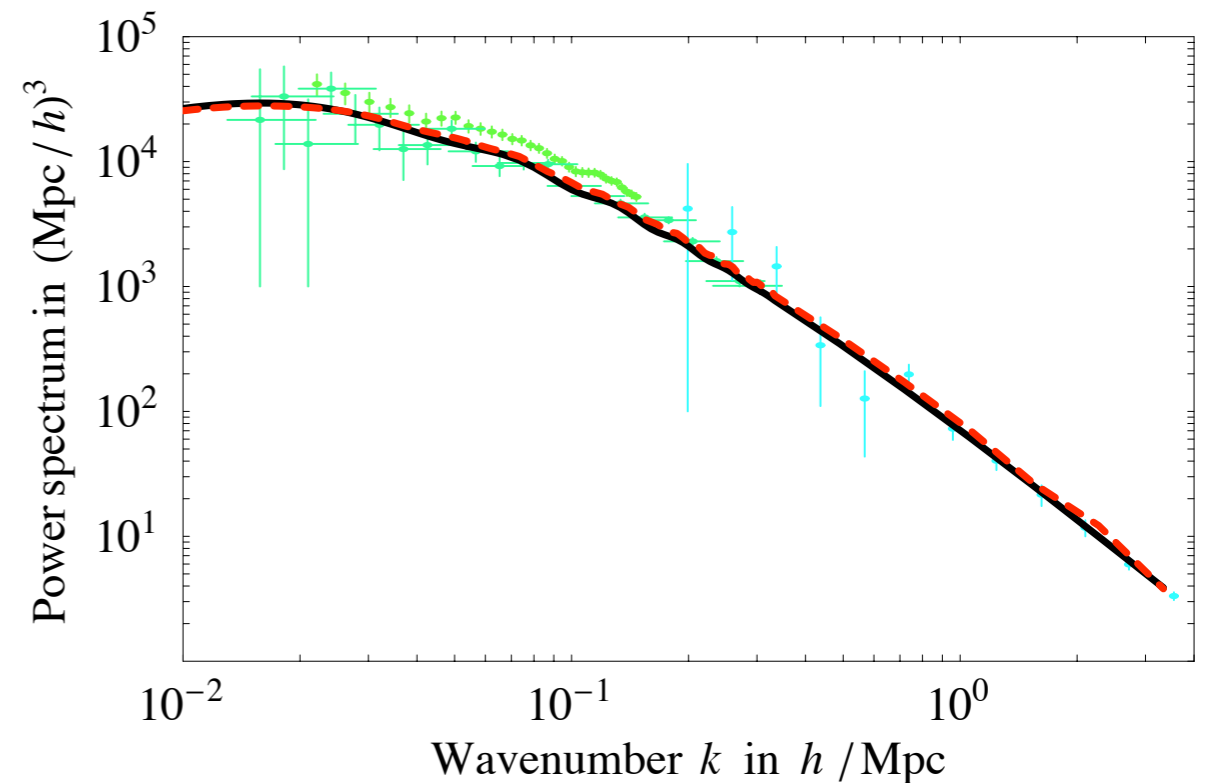
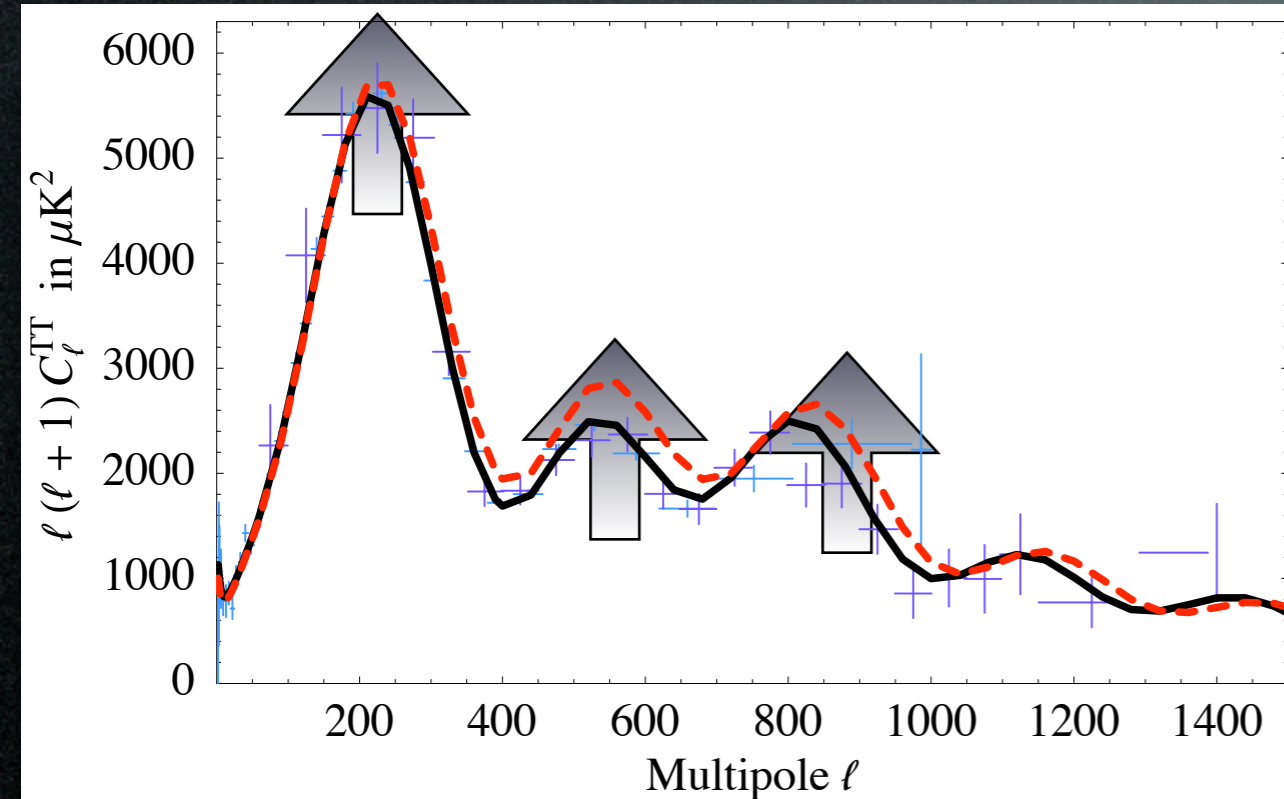
$N_\nu^{\text{int}} + \frac{4}{7}N_\phi$ interacting particles contribute to Rel energy density.

Results

Can some neutrinos be **sticky**?

$$N_{\nu}^{\text{norm}} \begin{cases} \text{red circle} \\ \text{green circle} \end{cases} \quad N_{\nu}^{\text{int}} \begin{cases} \text{blue circle} \\ \text{grey circle} \end{cases} \begin{cases} \text{wavy line} \\ \text{square} \end{cases} \bigg\} N_{\phi}$$

— 3 free-streaming ν
 - - - 3 interacting ν



Sticky neutrinos don't stream out of gravitational wells: contribute power to CMB.
 For massless neutrinos the effect on $P(k)$ is minor.

Quantitatively: (Friedland et al. 2007)

$$\left\{ \frac{\Delta C_{\ell}}{C_{\ell}}, \Delta \ell \right\} \approx - \{0.53, 57\} \frac{\rho_{\text{free}}}{\rho_{\text{free}} + \rho_{\text{sticky}} + \rho_{\gamma}}$$

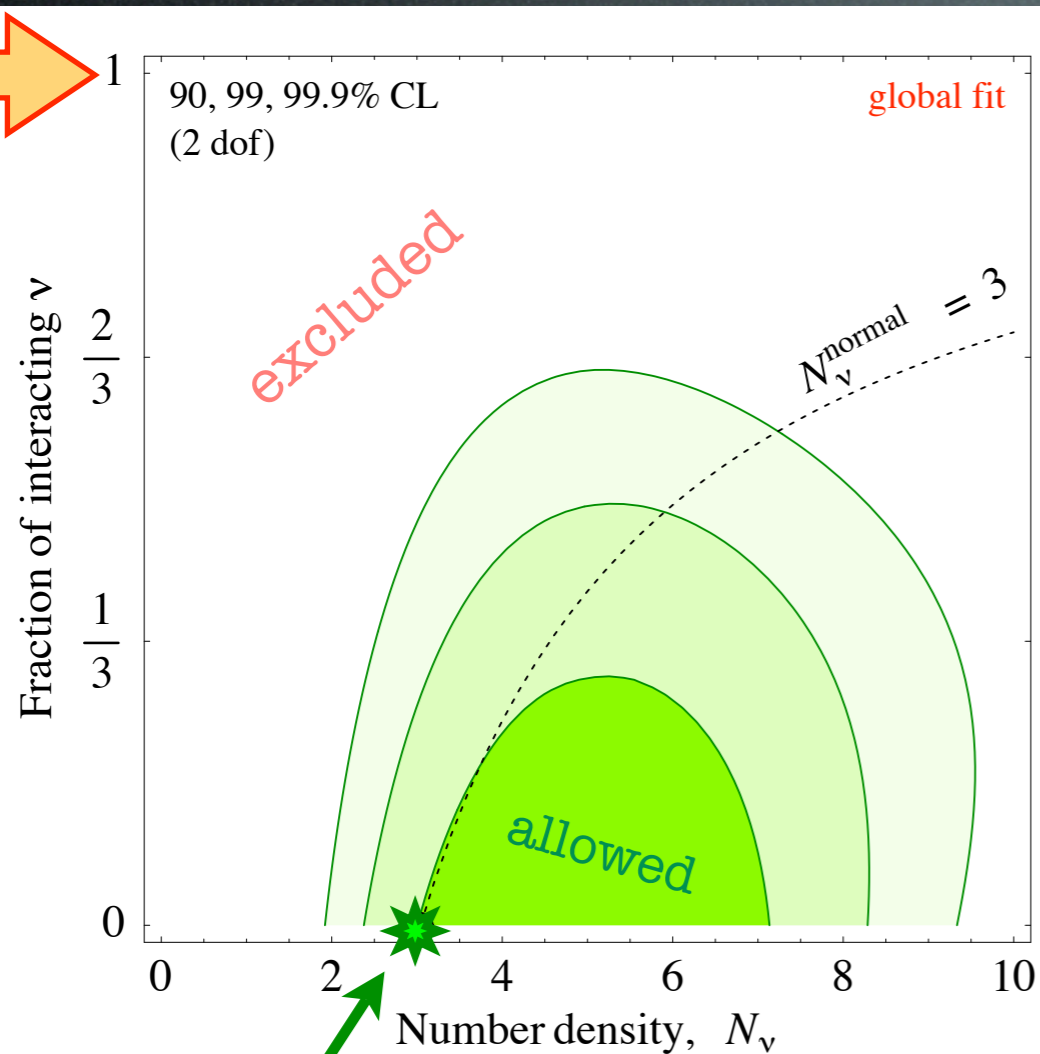
[Hannestad, JCAP 2005]

[Bell, Pierpaoli, Sigurdson, PRD73 (2006)]

Results

Can some neutrinos be **sticky**?

$$N_{\nu}^{\text{norm}} \begin{cases} \text{red circle} \\ \text{green circle} \end{cases} \quad N_{\nu}^{\text{int}} \begin{cases} \text{blue circle} \\ \text{grey circle} \end{cases} \begin{cases} \text{wavy line} \\ \text{square} \end{cases} \bigg\} N_{\phi}$$



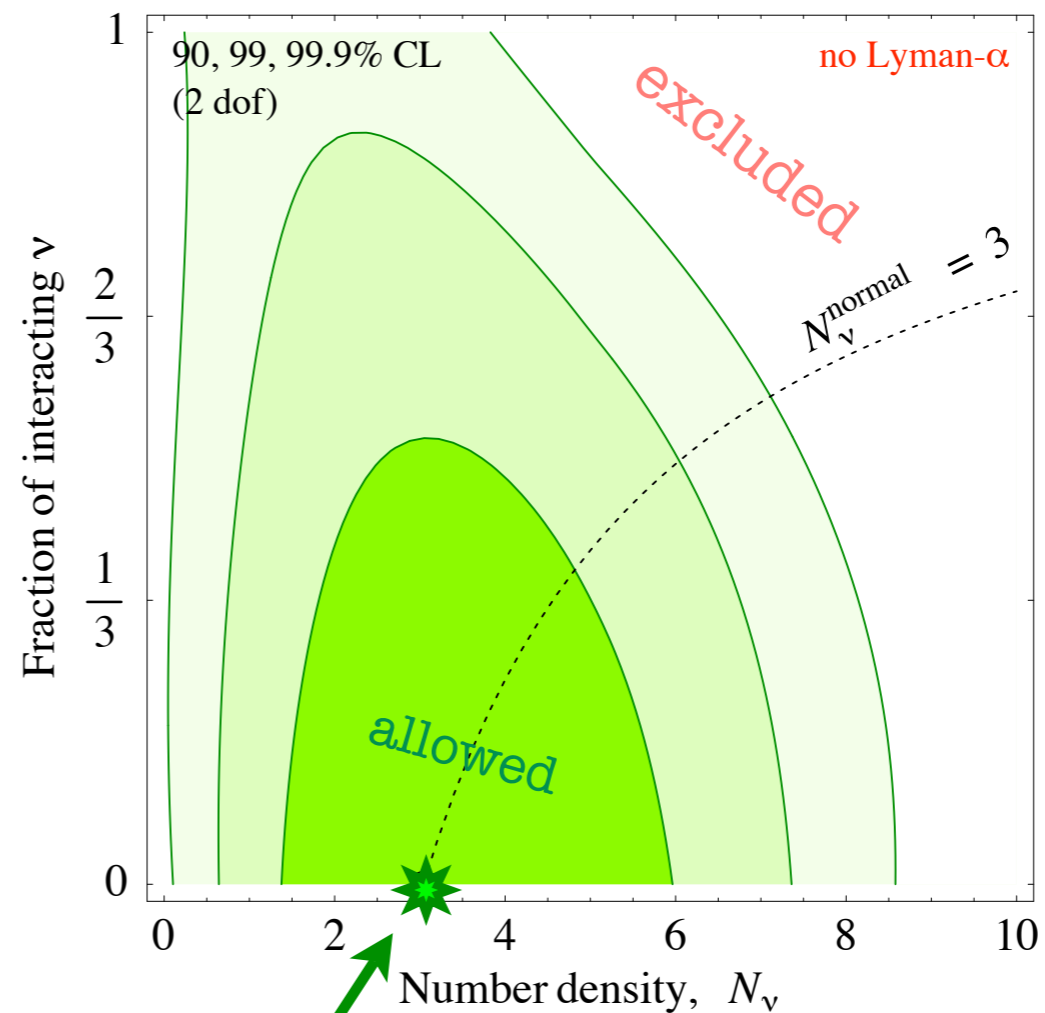
Standard cosmology

~ 1 sticky ν allowed

3 sticky ν excluded

(@ 99% CL,
global fit)

(at 5σ)



Standard cosmology

[see also Bell, Pierpaoli,
Sigurdson, PRD73 (2006)]

PLANCK will greatly improve
(will test 1 sticky at 4)

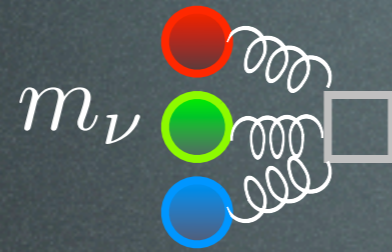
[Friedland, Zurek,
Bashinsky(2007)]

Results

[plots]

Can **all** neutrinos be **sticky**?

Massive neutrinos
and massless boson:



e.g. Neutrinoless Universe

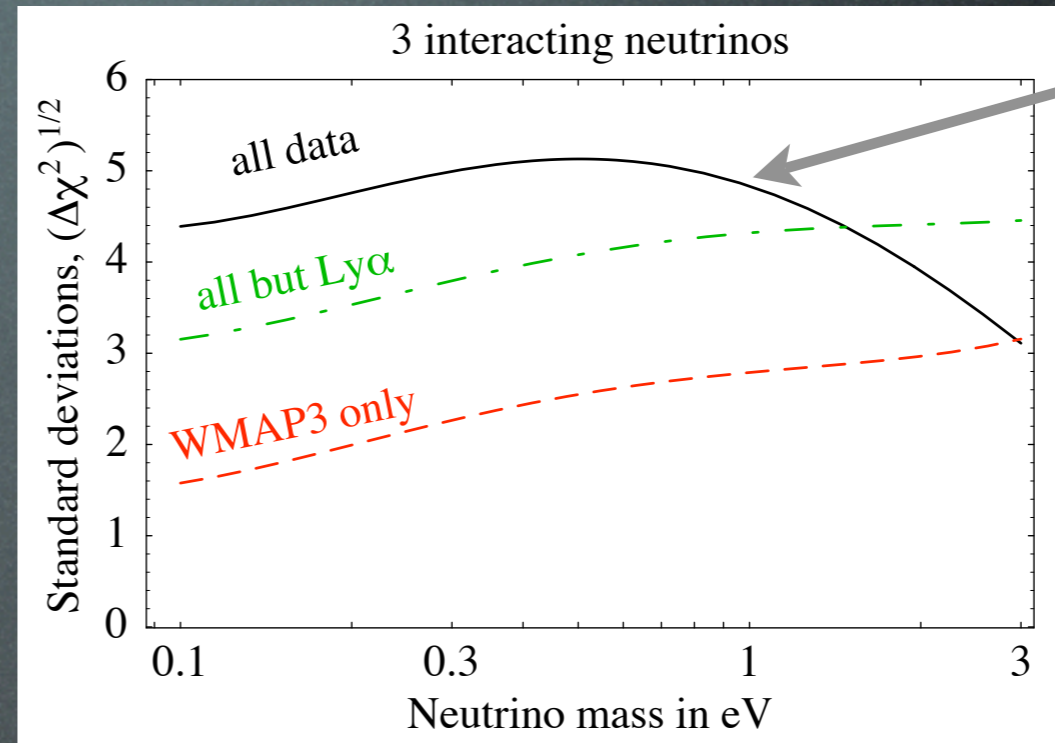
[Beacom et al. PRL '04]

e.g. Mass Varying Neutrinos,

[Fardon et al. JCAP '04]

Late Neutrino mass models

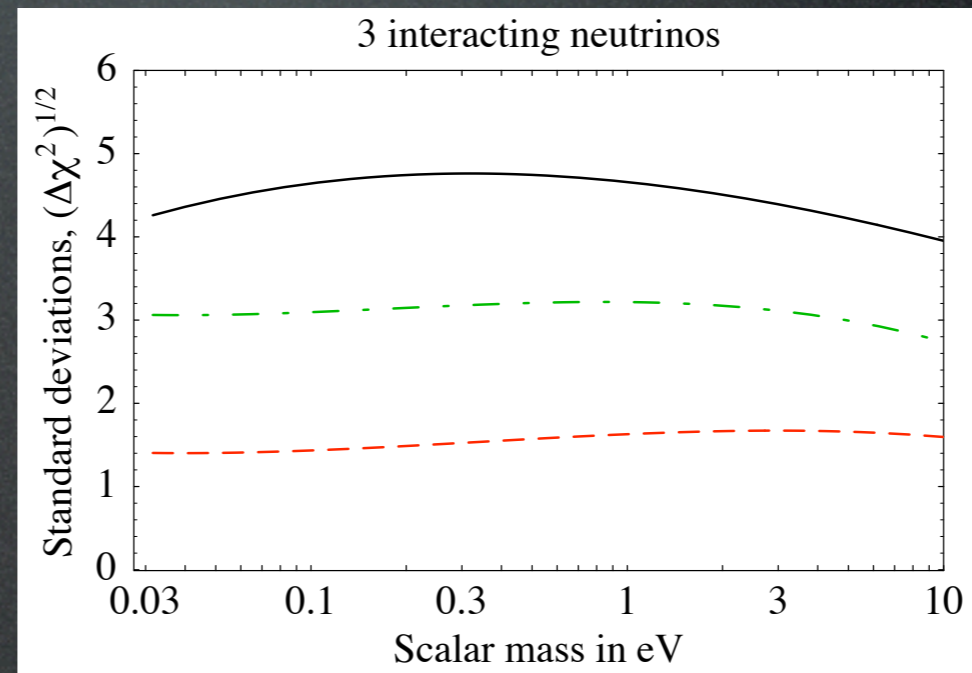
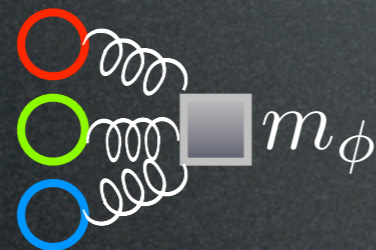
[Chacko et al. PRD '04]



disfavored at
3 to 5 σ

[see also Hannestad,
JCAP 2004]

Massless neutrinos
and massive boson:



Bottom Line:

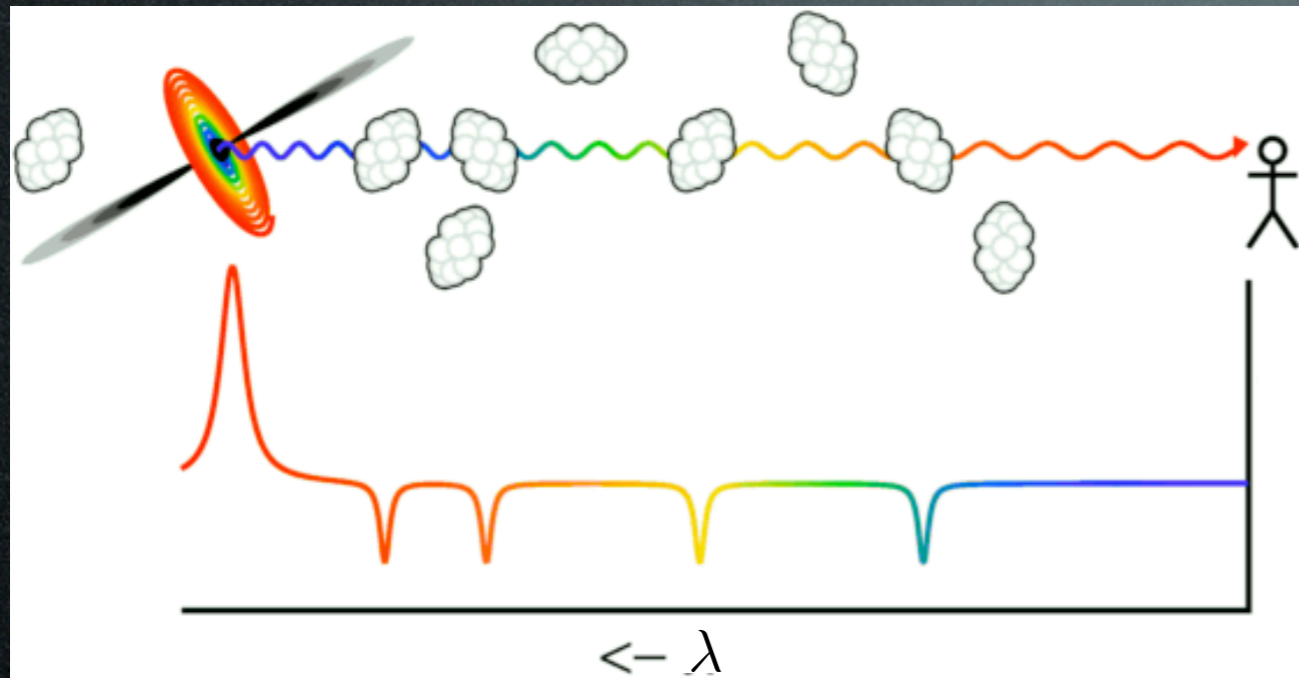
Cosmology **strongly disfavors**
fully interacting (non-freely streaming) neutrinos.

Conclusions & Messages

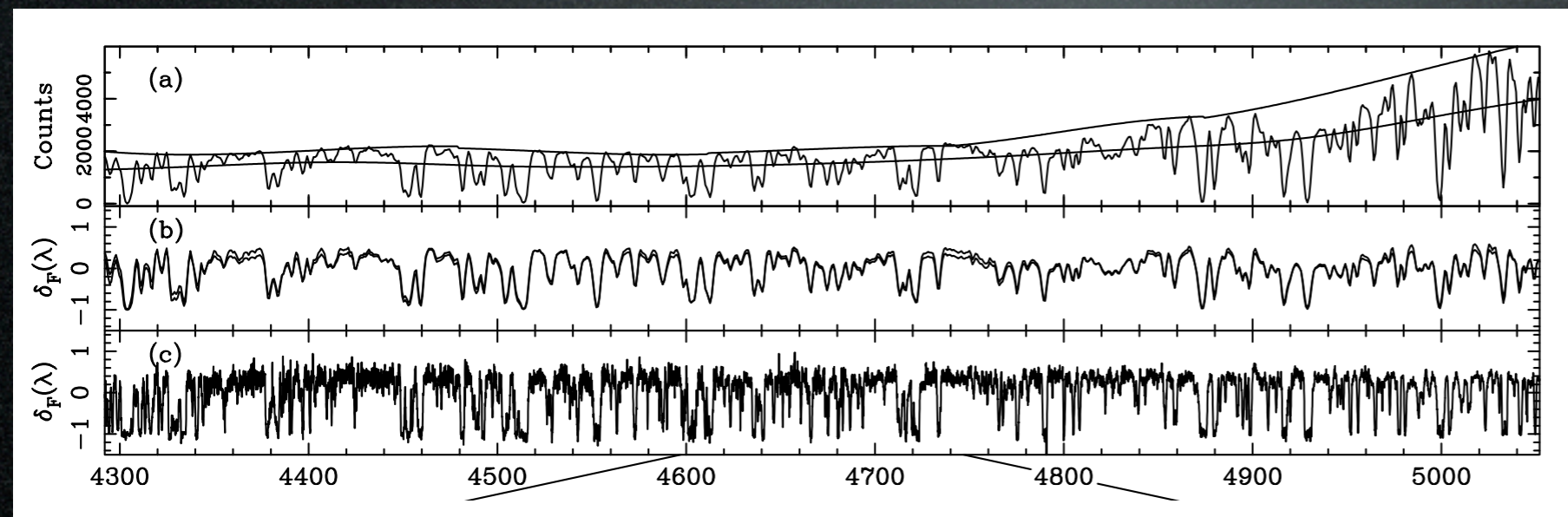
- Cosmology is a sensitive probe of neutrinos and possible new light particles; let's put at work the formalism (and a new code) for cosmological perturbation to extract the most from the full cosmo dataset.
- Cosmology gives **dominant bound on** $\sum m_{\nu_i}$; $\sum m_{\nu_i} < 0.40 \text{ eV}$ (global fit, 99.9% C.L.)
the bound tightens combining relatively less safe datasets.
- Cosmology seems to suggest **5 neutrinos** (2 extra);
but Ly-alpha are mainly driving the suggestion.
The massive extra neutrino of **LSND** was already **strongly disfavored**.
- Cosmology disfavors at various degrees neutrino interactions and other light particles: neutrinos **ought to free-stream**.
- Future observations will be powerful probes.

Extra slides

Lyman-alpha forest



Distant **quasar light**, redshifted and absorbed at Ly- α frequency by intervening matter, allows to **reconstruct matter distribution** along the line of sight.
But: **systematics** and uncertainties



Skepticism on Lyman- α : - very complicated measurement and analysis (from flux to matter spectra), different groups disagree (even on same data)
- non linearities
- HMD simulations don't include neutrinos

[back]

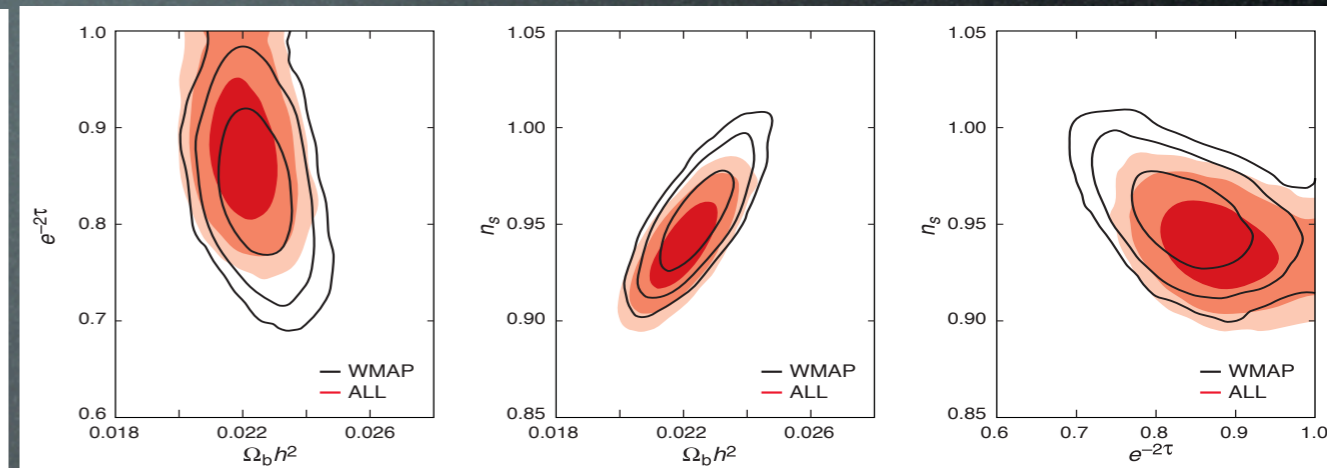
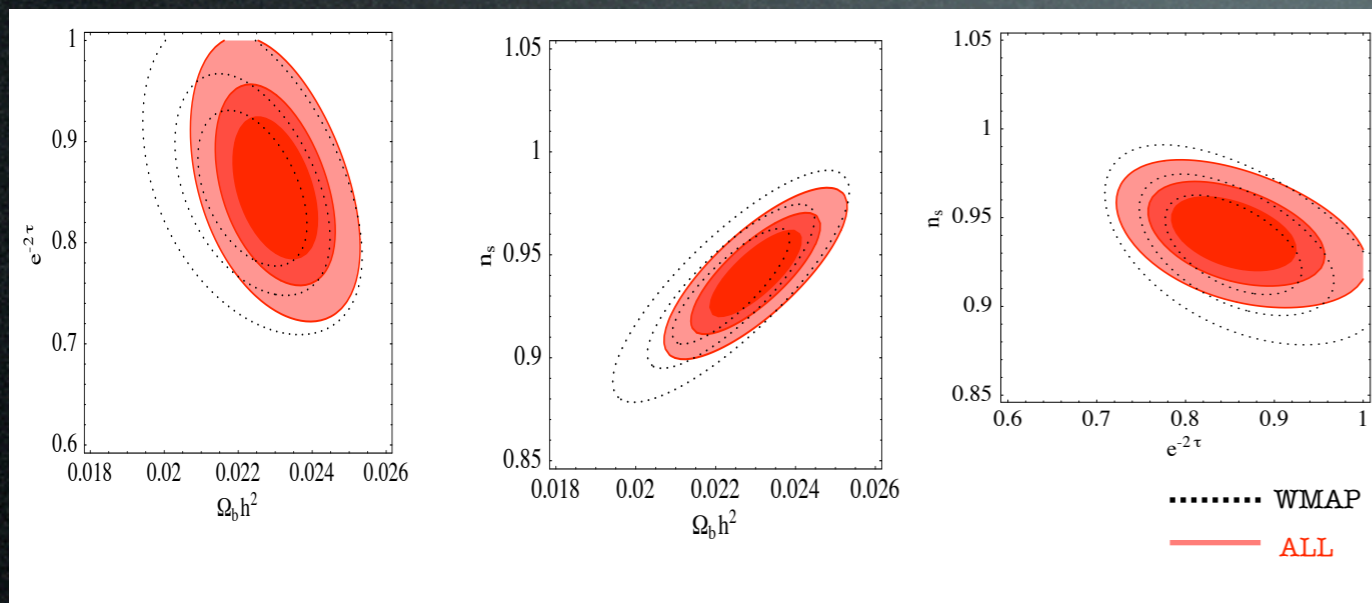
Comparing our code

[\[back\]](#)

Our analysis:



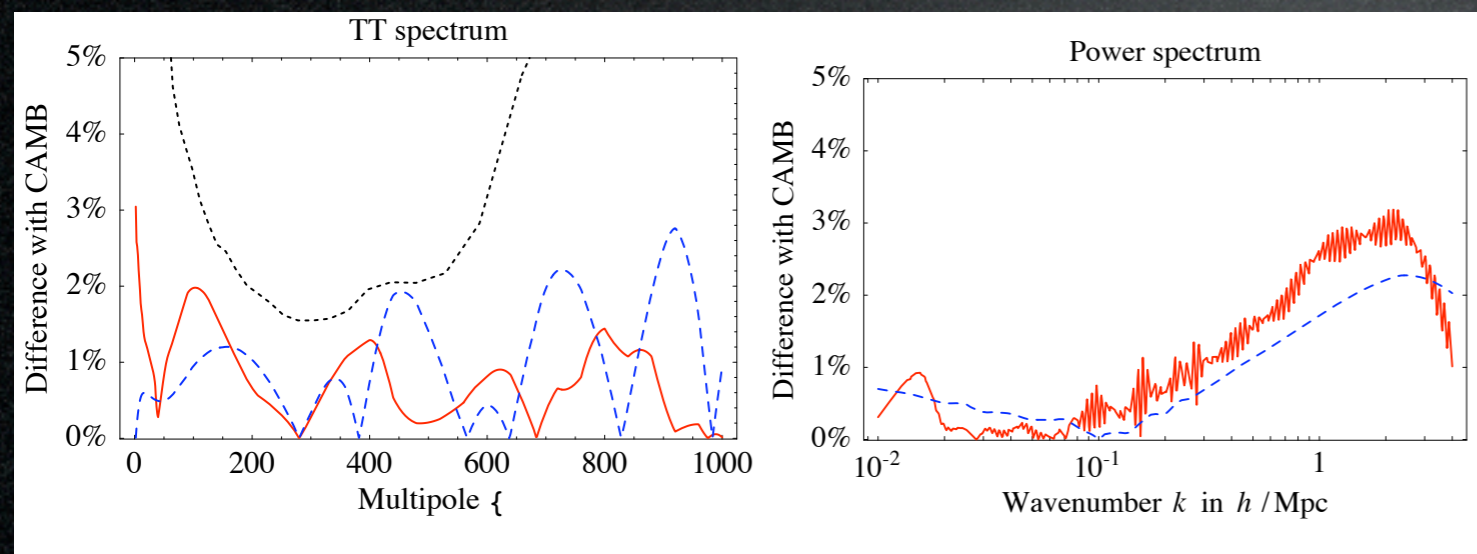
WMAP Science Team analysis:



[Spergel et al. WMAP 3yr results '05]

fit	A_s	h	n_s	τ	$100\Omega_b h^2$	$\Omega_{DM} h^2$
WMAP3	0.80 ± 0.05	0.704 ± 0.033	0.935 ± 0.019	0.081 ± 0.030	2.24 ± 0.10	0.113 ± 0.010
Global	0.84 ± 0.04	0.729 ± 0.013	0.951 ± 0.012	0.121 ± 0.025	2.36 ± 0.07	0.117 ± 0.003

Parameter	WMAP Only	WMAP+ SDSS	WMAP+ LRG	WMAP + SN Gold
$100\Omega_b h^2$	$2.233^{+0.072}_{-0.091}$	$2.233^{+0.062}_{-0.086}$	$2.242^{+0.062}_{-0.084}$	$2.227^{+0.065}_{-0.082}$
$\Omega_m h^2$	$0.1268^{+0.0073}_{-0.0128}$	$0.1329^{+0.0057}_{-0.0109}$	$0.1337^{+0.0047}_{-0.0098}$	$0.1349^{+0.0054}_{-0.0106}$
h	$0.734^{+0.028}_{-0.038}$	$0.709^{+0.024}_{-0.032}$	$0.709^{+0.016}_{-0.023}$	$0.701^{+0.020}_{-0.026}$
A	$0.801^{+0.043}_{-0.054}$	$0.813^{+0.042}_{-0.052}$	$0.816^{+0.042}_{-0.049}$	$0.827^{+0.045}_{-0.053}$
τ	$0.088^{+0.028}_{-0.034}$	$0.079^{+0.029}_{-0.032}$	$0.082^{+0.028}_{-0.033}$	$0.079^{+0.028}_{-0.034}$
n_s	$0.951^{+0.015}_{-0.019}$	$0.948^{+0.015}_{-0.018}$	$0.951^{+0.014}_{-0.018}$	$0.946^{+0.015}_{-0.019}$
σ_8	$0.744^{+0.050}_{-0.060}$	$0.772^{+0.036}_{-0.048}$	$0.781^{+0.032}_{-0.045}$	$0.784^{+0.035}_{-0.049}$
Ω_m	$0.238^{+0.027}_{-0.045}$	$0.266^{+0.025}_{-0.040}$	$0.267^{+0.017}_{-0.029}$	$0.276^{+0.022}_{-0.036}$



agreement is at **few %** level and within current precision of data

Neutrinos in the Cosmo

LEPTONS

Neutrino Properties

SUM OF THE NEUTRINO MASSES, m_{tot}

(Defined in the above note), of effectively stable neutrinos (i.e., those with mean lives greater than or equal to the age of the universe). These papers assumed Dirac neutrinos. When necessary, we have generalized the results reported so they apply to m_{tot} . For other limits, see SZALAY 76, VYSOTSKY 77, BERNSTEIN 81, FREESE 84, SCHRAMM 84, and COWSIK 85.

VALUE (eV)	CL%	DOCUMENT ID	TECN	COMMENT
• • • We do not use the following data for averages, fits, limits, etc. • • •				
< 0.24	95	54 CIRELLI	06 COSM	
< 0.62	95	55 HANNESTAD	06 COSM	
< 0.52	95	56 KRISTIANSEN	06 COSM	
< 0.17	95	54 SELJAK	06 COSM	
< 2.0	95	57 ICHIKAWA	05 COSM	
< 0.75		58 BARGER	04 COSM	
< 1.0		59 CROTTY	04 COSM	
< 0.7		60 SPERGEL	03 COSM	WMAP
< 0.9		61 LEWIS	02 COSM	
< 4.2		62 WANG	02 COSM	CMB
< 2.7		63 FUKUGITA	00 COSM	
< 5.5		64 CROFT	99 ASTR	Ly α power spec
<180		SZALAY	74 COSM	
<132		COWSIK	72 COSM	
<280		MARX	72 COSM	
<400		GERSHTEIN	66 COSM	

(from Particle Data Book 2008)

Number of Neutrino Types

The neutrinos referred to in this section are those of the Standard $SU(2) \times U(1)$ Electroweak Model possibly extended to allow nonzero neutrino masses. Light neutrinos are those with $m < m_Z/2$. The limits are on the number of neutrino mass eigenstates, including ν_1 , ν_2 , and ν_3 .

Limits from Astrophysics and Cosmology

Number of Light ν Types

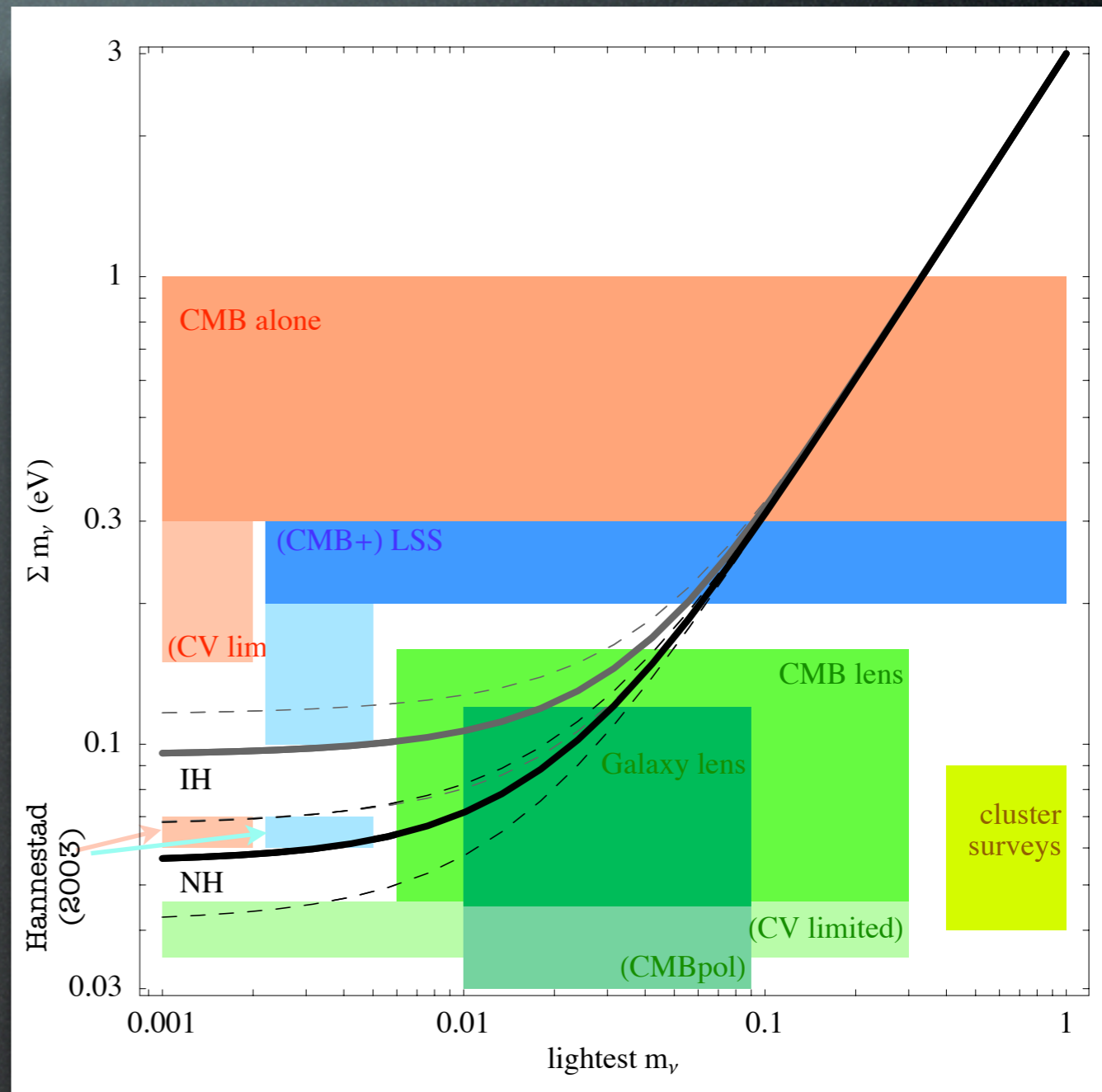
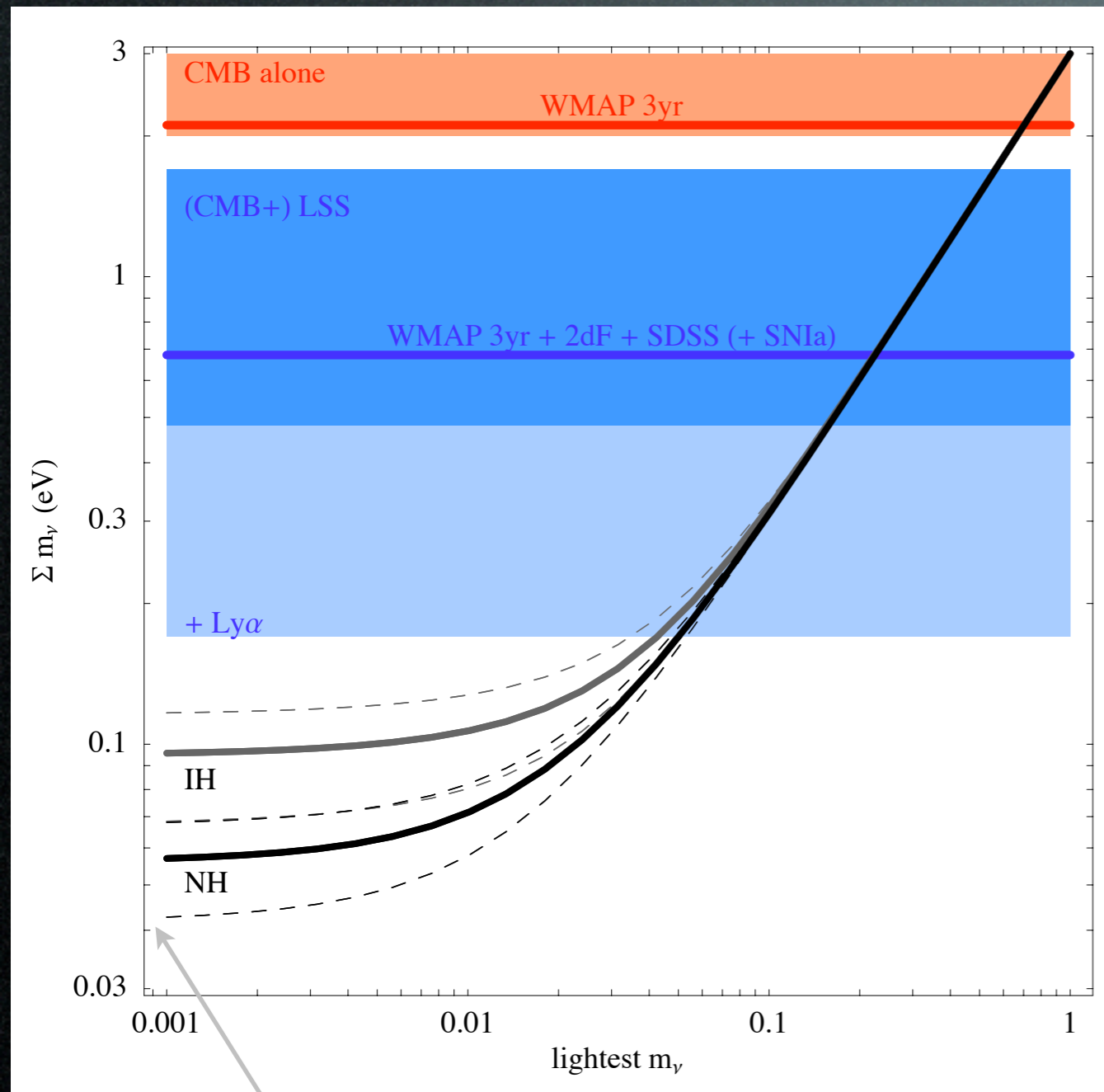
("light" means $<$ about 1 MeV). See also OLIVE 81. For a review of limits based on Nucleosynthesis, Supernovae, and also on terrestrial experiments, see DENEGRİ 90. Also see "Big-Bang Nucleosynthesis" in this Review.

VALUE	CL%	DOCUMENT ID	TECN	COMMENT
• • • We do not use the following data for averages, fits, limits, etc. • • •				
$3 < N_\nu < 7$	95	3 CIRELLI	06 COSM	
$2.7 < N_\nu < 4.6$	95	4 HANNESTAD	06 COSM	
$3.6 < N_\nu < 7.4$	95	3 SELJAK	06 COSM	
< 4.4		5 CYBURT	05 COSM	
< 3.3		6 BARGER	03C COSM	
$1.4 < N_\nu < 6.8$		7 CROTTY	03 COSM	
$1.9 < N_\nu < 6.6$		7 PIERPAOLI	03 COSM	
$2 < N_\nu < 4$		LISI	99	BBN
< 4.3		OLIVE	99	BBN
< 4.9		COPI	97	Cosmology
< 3.6		HATA	97B	High D/H quasar abs.
< 4.0		OLIVE	97	BBN; high ^4He and ^7Li
< 4.7		CARDALL	96B	COSM High D/H quasar abs.
< 3.9		FIELDS	96	COSM BBN; high ^4He and ^7Li
< 4.5		KERNAN	96	COSM High D/H quasar abs.
< 3.6		OLIVE	95	BBN; ≥ 3 massless ν
< 3.3		WALKER	91	Cosmology

On neutrino masses

present bounds

future sensitivities



$$\approx \sqrt{\Delta m_{\text{atm}}^2}$$

Legenda: the bound or measurement will fall somewhere in the colored box; “where it’ll fall exactly” depends on the author, the experiment considered, priors, the weather...

best summary reference: Lesgourgues, Pastor review

On neutrino masses

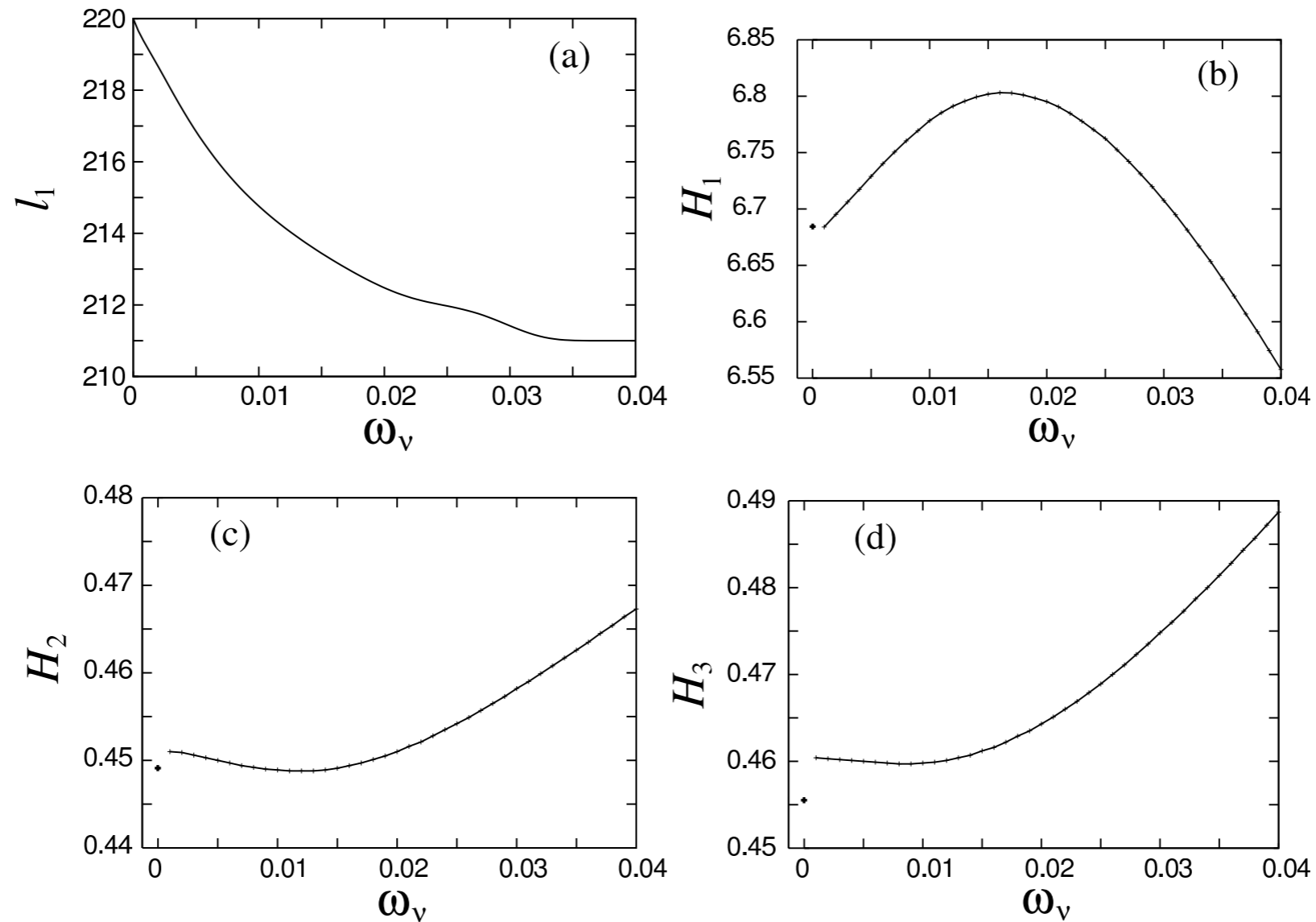


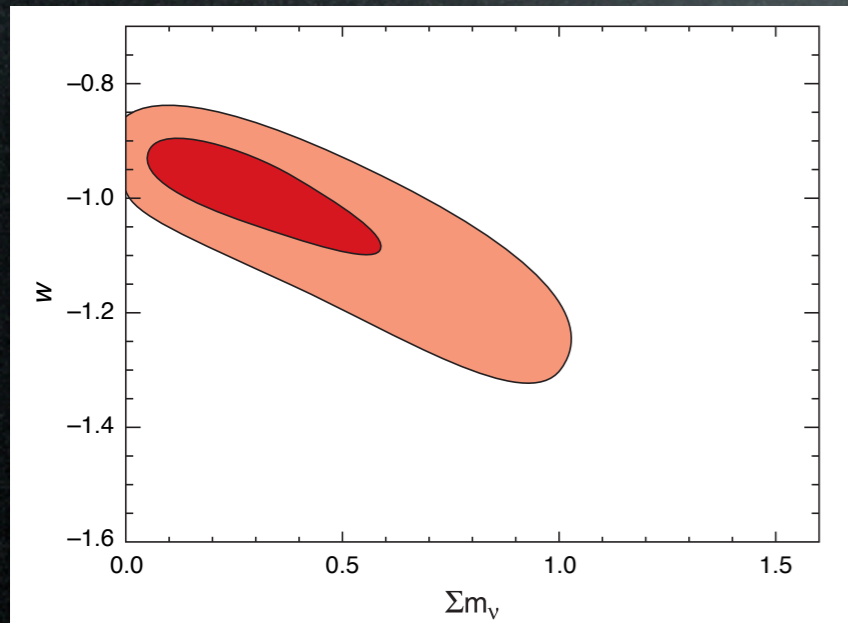
FIG. 5: Response of the four reduced CMB observables to the variation of ω_ν . The isolated points show the values at $\omega_\nu = 0$, which do not connect to the $\omega_\nu \neq 0$ values smoothly.

Degeneracies

m_ν effect can be cancelled
by $w < -1$.

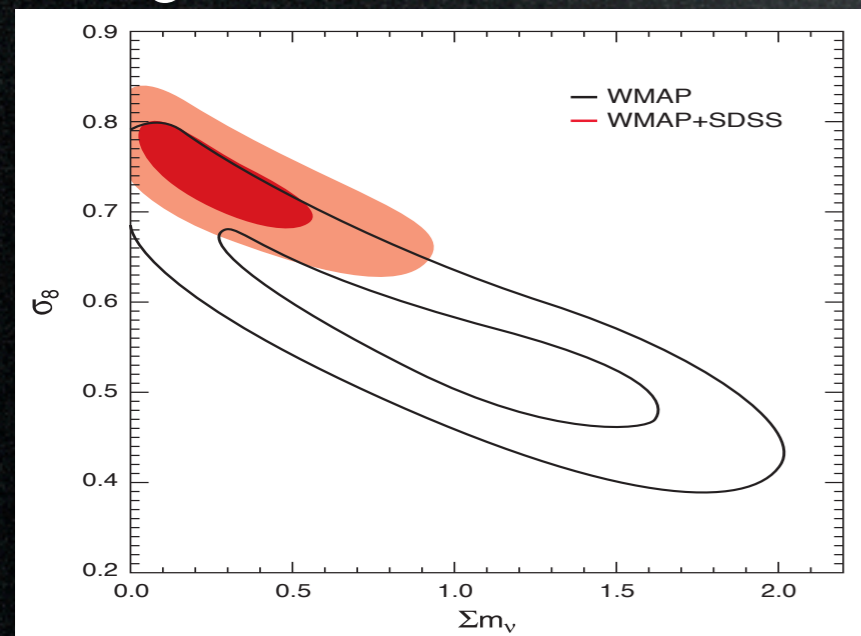
Hannestad,
astro-ph/0505551

(SNIa data allow less Ω_Λ , hence more Ω_m ,
if $w < -1$; more Ω_m brings back up the $P(k)$)

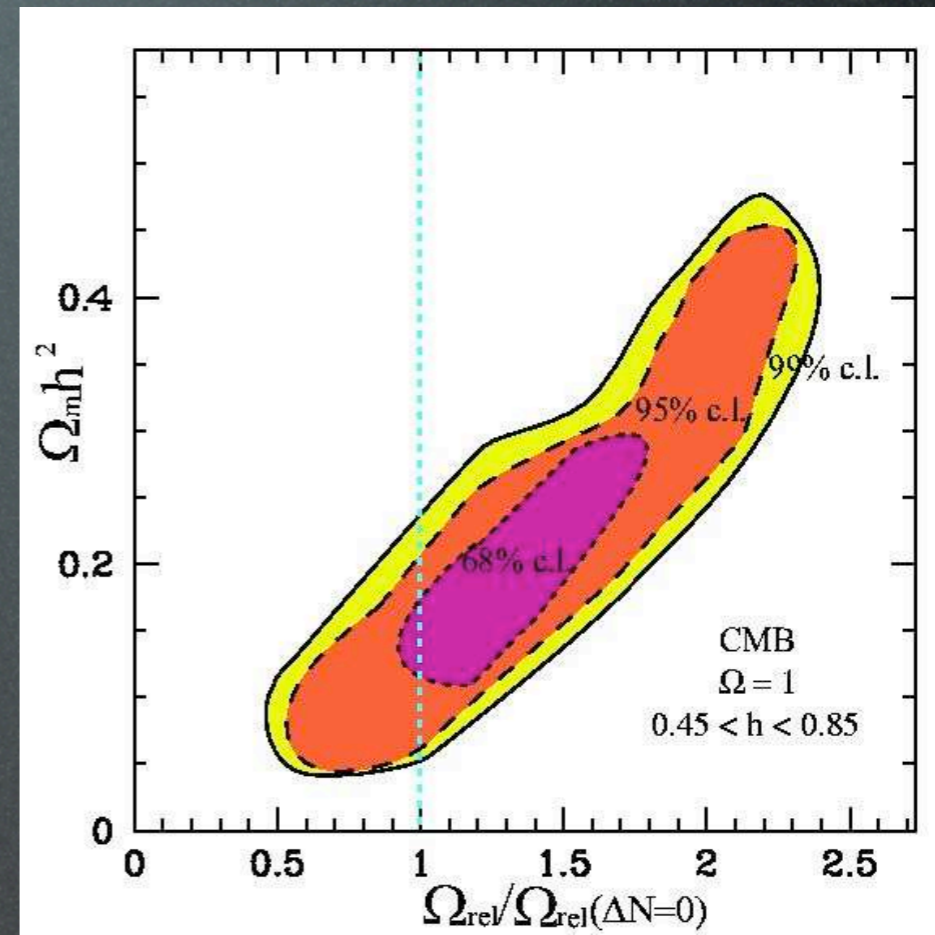


WMAP 3yr, Spergel et al.

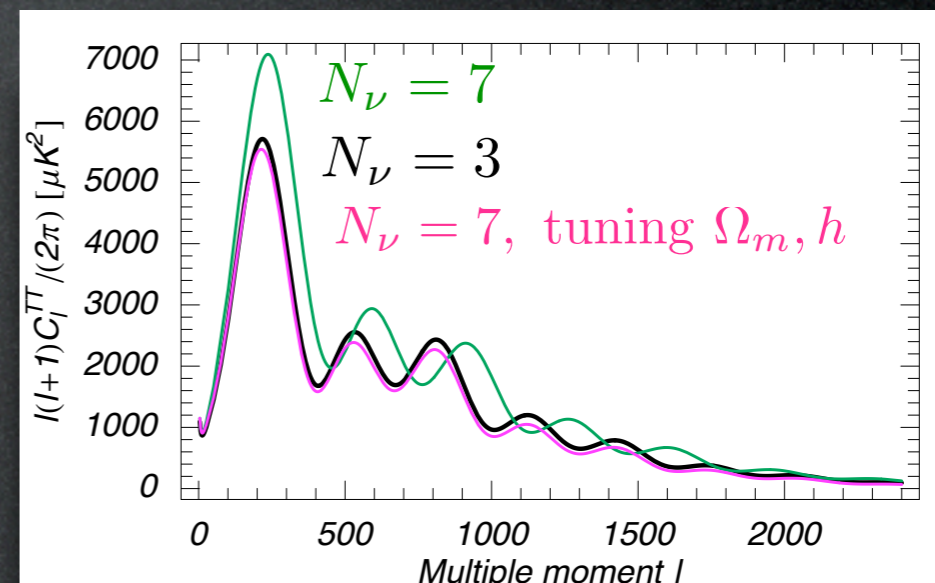
or by low σ_8



Large N_ν can be cancelled
by large Ω_m or h



Bowen, Hansen, Melchiorri, Silk, Trotta,
MNRAS 334 (2002)



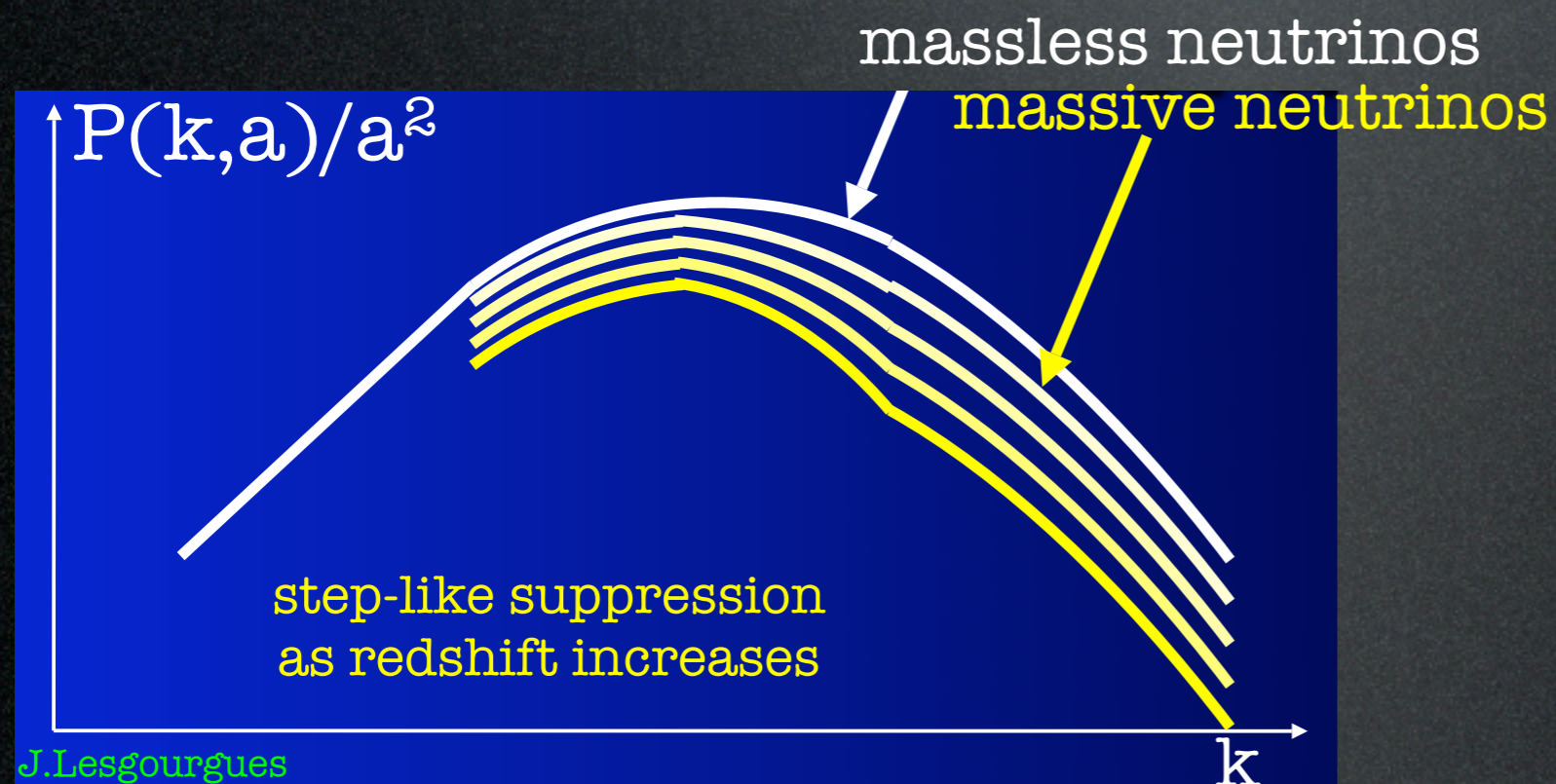
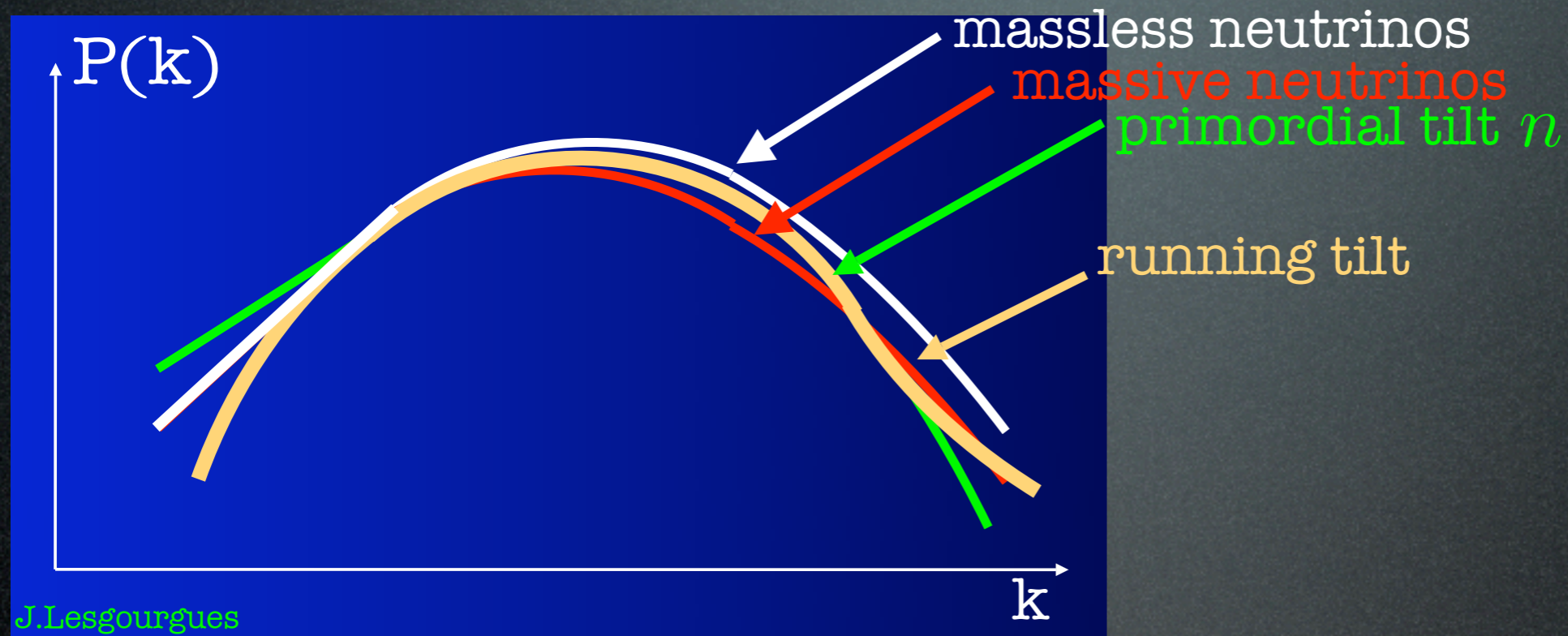
Friedland et al. 2007

[back to Nnu]

Degeneracies

$\sum m_\nu$ will **not** be forever degenerate with other parameters:

Julien Lesgourgues, talks in 2007

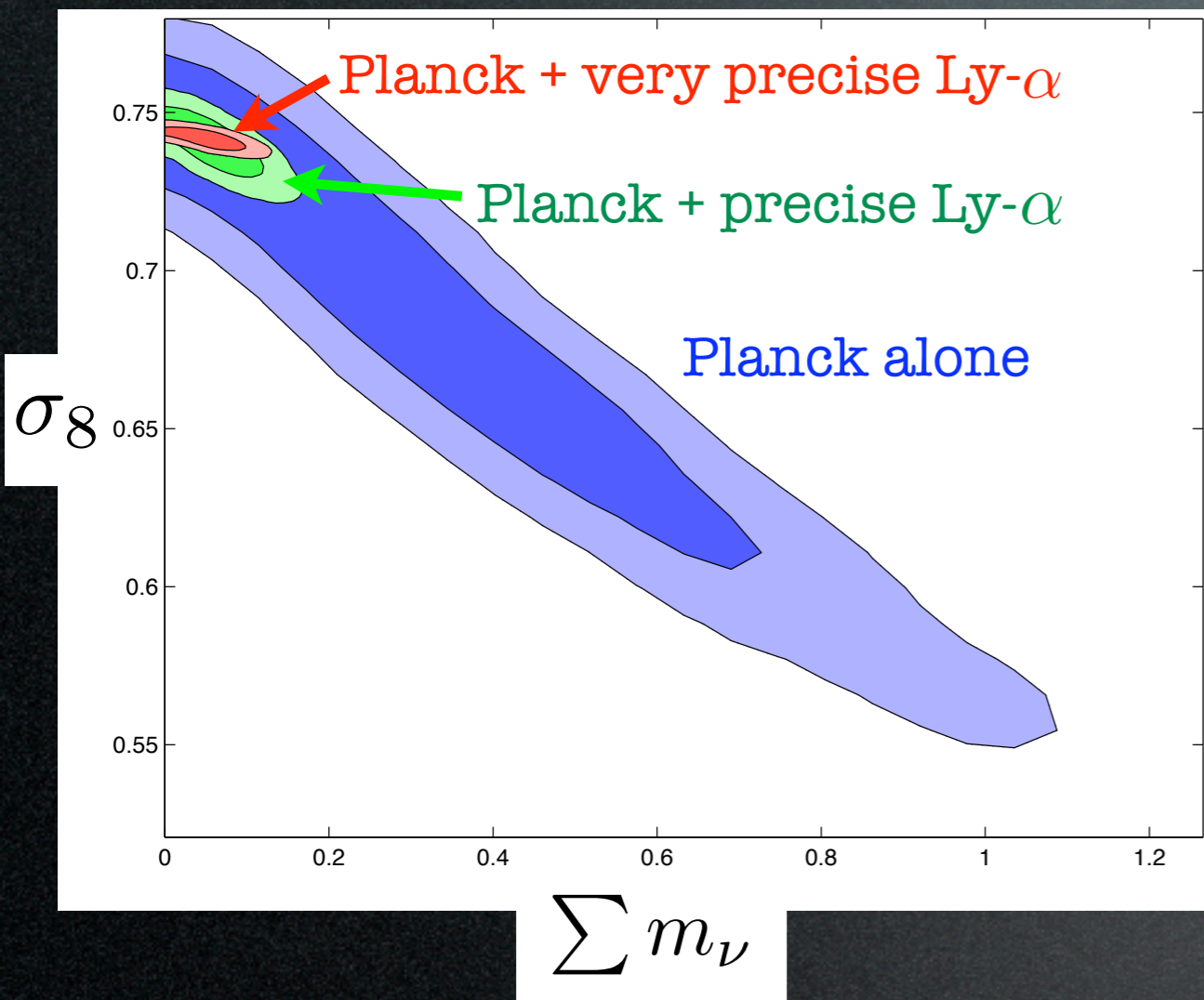


Degeneracies

$\sum m_\nu$ will **not** be forever degenerate with other parameters:

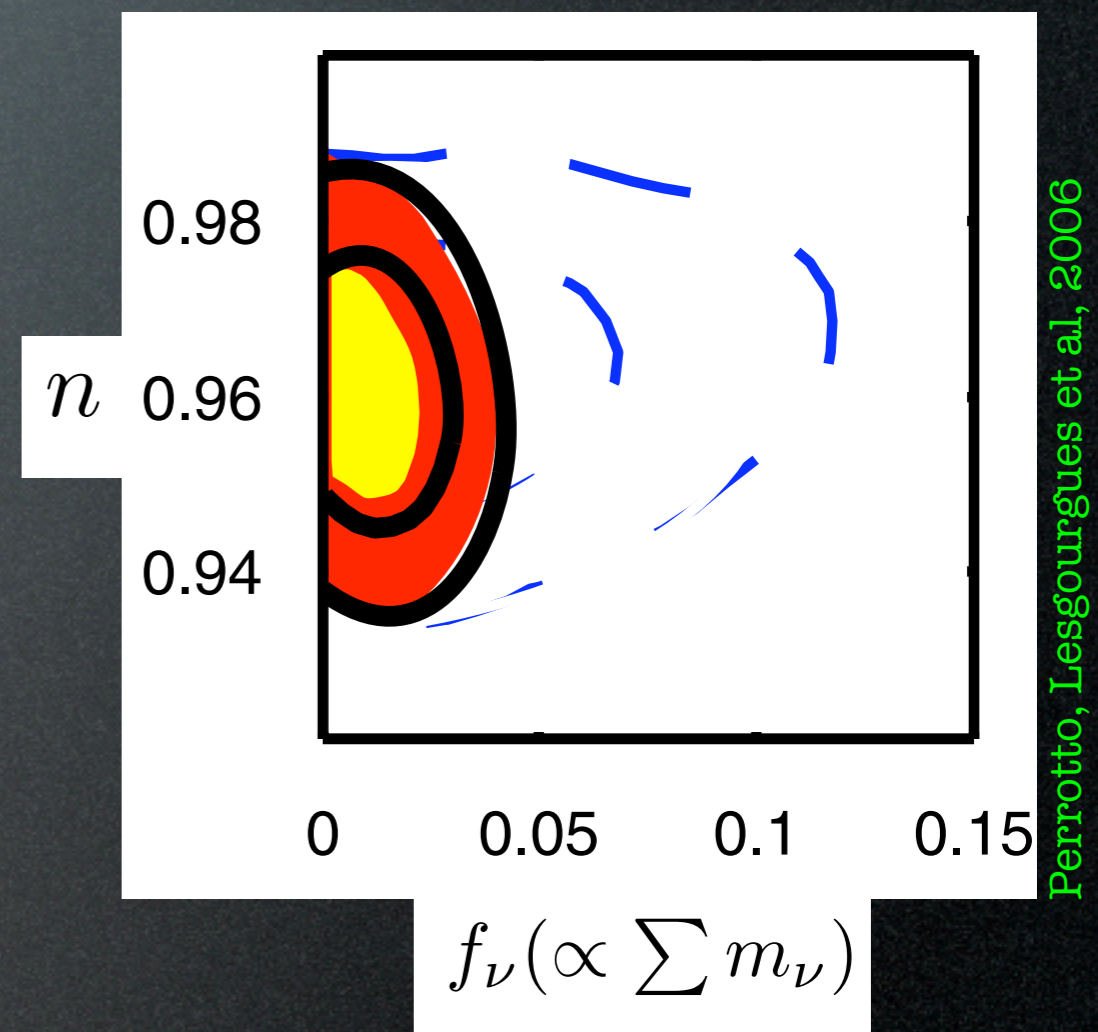
Julien Lesgourgues, talks in 2007

Planck + precision Ly- α :



Gratton, Lewis, Efstathiou 2007

Planck (with lensing extraction):



Perrotto, Lesgourgues et al, 2006